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مجلة جامعة بني وليد للعلوم الإنسانية والتطبيقية - العدد السادس و العشرون - ديسمبر 2022 م

# مجلة جامعة بني وليد
## للعلوم الإنسانية و التطبيقية

مجلة فصلية محكمة تصدر عن جامعة بني وليد

من محتويات العدد :



السنة السادسة العدد السادس والعشرون المجلد الأول ديسمبر 2022

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بني وليد – ليبيا

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المجلد الأول– ديسمبر 2022 م

التوثيق: الـدار الوطنية للكتاب بنغازي 121 /2017

## مجلة جامعة بني وليد للعلوم الإنسانية والتطبيقية

## السنة السادسة – العدد السادس والعشرون –

## المجلد الأول– ديسمبر 2022 م

## قواعد النشر بمجلة جامعة بني وليد للعلوم الإنسانية والتطبيقية

***مجلة جامعة بني وليد للعلوم الإنسانية والتطبيقية***

مجلة علمية فصلية محكمة تهتم بنشر البحوث والدراسات العلمية الأصيلة والمبتكرة في العلوم الإنسانية والتطبيقية .

***وإذ تـرحب المجلة*** بالإنتاج المعرفي والعلمي للباحثين في المجالات المشار إليها تحيطكـم علماً بقواعد النشر بها وهي كالتالي :

1– تقبل البحوث باللغتين العربية والإنجليزية على أن تعالج القضايا والموضوعات بأسلوب علمي موثق يعتمد الإجرائية المعتمدة في الأبحاث العلمية، وذلك بعرض موضوع الدراسة وأهدافها ومنهجها وتقنياتها وصولاً إلى نتائجها وتوصياتها ومقترحاتها.

2– يكون التوثيق بذكر المصادر والمراجع بأسلوب أكاديمي يتضمن:

أ– الكتب : اسم المؤلف، عنوان الكتاب، مكان وتاريخ النشر، اسم الناشر، رقم الصفحة .

ب– الدوريات : اسم الباحث، عنوان البحث، اسم المجلة، العدد وتاريخه، رقم الصفحة .

3– معيار النشر هو المستوى العلمي والموضوعية والأمانة العلمية ودرجة التوثيق وخلو البحث من الأخطاء التحريرية واللغوية وأخطاء الطباعة.

4– أن يكون النص مطبوعا على برنامج ( **Microsoft Word** ) ويكون حجم الخط (**14**) ونوعه (**Simplified Arabic**)، على حجم ورق **A4** .

5–أن لا يزيد حجم الدراسة أو البحث على (25) صفحة كحد اقصى وان يرفق بخلاصة للبحث أو المقالة لا تتجاوز(60)كلمة تنشر معه عند نشره .

6– ترحب المجلة بتغطية المؤتمرات والندوات عبر تقارير لا تتعدى (10) صفحات (A4) كحد اقصى، يذكر فيها مكان الندوة أو المؤتمر وزمانها وأبرز المشاركين، مع رصد أبرز ما جاء في الأوراق والتعقيبات والتوصيات .

7– ترحب المجلة بنشر مراجعات الكتب بحدود (10) صفحات (A4) كحد أقصى على أن لا يكون قد مضى على صدور الكتاب أكثر من عامين. على أن تتضمن المراجعة عنوان الكتاب وأسم المؤلف ومكان النشر وتاريخه وعدد الصفحات، وتتألف المراجعة من عرض وتحليل ونقد، و أن تتضمن المراجعة خلاصة مركزة لمحتويات الكتاب، مع الاهتمام بمناقشة أطروحات المؤلف ومصداقية مصادره وصحة استنتاجاته .

8– يرفق مع كل دراسة أو بحث تعريف بالسيرة الأكاديمية والدرجة العلمية والعمل الحالي للباحث .

9– لا تدفع المجلة مكافآت مالية عما تقبله للنشر فيها .

10– لا تكون المواد المرسلة للنشر في المجلة قد نشرت أو أرسلت للنشر في مجلات أخرى.

11– تخضع المواد الواردة للتقييم، وتختار هيئة تحرير المجلة (سرياً) من تراه مؤهلاً لذلك، ولاتعاد المواد التي لم تنشر إلى أصحابها.

12 – يتم إعلام الباحث بقرار التحكيم خلال شهرين من تاريخ الإشعار باستلام النص، وللمجلة الحق في الطلب من الباحث أن يحذف أي جزء أو يعيد الصياغة،بما يتوافق وقواعدها.

13– تحتفظ المجلة بحقها في نشر المادة وفق خطة التحرير، وتؤول حقوق الطبع عند إخطار الباحث بقبول بحثه للنشر للمجلة دون غيرها.

14– مسؤولية مراجعة و تصحيح و تدقيق لغة البحث تقع علي الباحث، على أن يقدم ما يفيد بمراجعة البحث لغويا، ويكون ذلك قبل تقديمه للمجلة .

15– ترسل البحوث والدراسات والمقالات باسم مدير التحرير.

بخصوص البحوث والدراسات والمقالات التي تسلم إلى مقر المجلة، فإن البحث يسلم على قرص مدمج(CD) مرفقا بعدد 2 نسخة ورقية .

**للمزيد من المعلومات والاستفسار يمكنكم المراجعة عبر :**

**البريد الإلكتروني**

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( مجلة جامعة بني وليد للعلوم الإنسانية و التطبيقية )

**مقر المجلة**

إدارة المكتبات والمطبوعات والنشر بالجامعة – المبنى الإداري

لجامعة بني وليد

بني وليد – ليبيا

## محتويات العدد

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# ANALYTICAL DETERMINATION OF BOUNDARY SHEAR STRESS OVER FIXED AND MOBILE BEDS FOR SUBMERGED WALL JETS

**Dr.Mohamed Ibrahim Ghoma**

**Faculty of Engineering, University of Gharyan**

**ABSTRACT**

This paper describes ananalytical model created for the turbulent wall jet-induced unstable scour hole. Also presented is the use of the analytical model for various wall jet situations. Estimates of Reynolds Stress at 1 cm above the bed at various streamwise locations for a rough bed were made using anAcoustic Doppler Velocimeter ADV.The results revealed that the region near the wall jet, where there is flow recirculation, is the part of the flow that is the most sensitive to Reynolds stressfor flows over both a fixed rough bed and a mobile bed. The analytical model matches the experimentaldata well in the region after x/b0 15, where the non-dimensional error was 0.0035.

## 1. INTRODUCTION

Accurate determination of the bed shear stress distribution is one of the most important requirements needed to predict local sediment transport rates and hence local scouring. Limited attention has so far been paid to the shear stress pattern in submerged wall-jets over fully rough beds and the way in which the shape of these beds have been changed by local scouring processes. The resulting movement of the bed material in the direction of flow can be estimated once the local applied shear stress has exceeded the local critical bed shear stress.The critical shear stress is referred to as the initial motion condition (Nalluri et al. 2001) and is estimated based on a ratio of the submerged grain weight the applied boundary shear stress, and the local bed slope following the ideas of Shields (1936). More recent work by Dey et al (2007) presented the Reynolds and boundary shear stresses in submerged jets on horizontal rough boundaries. They measured the flow in submerged jets on horizontal rough boundaries with an Acoustic Doppler Velocimeter (ADV). Their results showed that the boundary shear stress in the vertical profiles increases with an increase in boundary roughness.

An early approach to predict the bed shear stress distribution over a rough fixed bed caused by a submerged wall jet is the formula developed by Hogg et al. (1997). They presented new scaling laws for the spatial variation of the bed shear stress of a two-dimensional turbulent wall jet flowing over a fixed rough boundary. These laws were then used in the development of an analytical framework to model the progressive erosion of an initially flat bed of grains by a submerged turbulent jet.Use of scaling laws for the downstream variation of the boundary shear stress then permits the calculation of the shape of the steady-state scour at different time steps. In this case, they applied the jet flow scaling over fixed rough boundaries to erodible boundaries, on the assumption that the aspect ratio, i.e. the ratio of the depth of erosion to streamwise extent of the scour hole, is small. The authors estimated the shear stress exerted by a diffusive jet by integrating the momentum equation for 1D flow on the assumption that hydrostatic pressure holds and streamwise variations of normal stress are negligible. This led to the streamwise rate of change of the momentum flux being described by the following equation:

$$\tau_b = -\frac{d}{dx}\int_0^\infty \rho u^2 \, dy \qquad (1)$$

Where $u$ represents the jet velocity varying along the longitudinal and vertical directions. This model requires that the mean flow does not separate from the boundary at any downstream location to avoid the need to introduce models of regions in which there is flow recirculation. The profile of the scour hole was calculated by defining a profile that leads to an equalling of the shear stress distribution along the bed surface to the critical shear stress for incipient motion. They calculated the critical value of the ratio $\tau_b/\Delta\rho g d$ for incipient motion on a bed with a longitudinal gradient:

$$\theta_c = \theta_{crit}\frac{\sin(\alpha+\beta)}{\sin\alpha} \qquad (2)$$

Where $\beta$is the bed–slope angle and $\alpha$is the angle of repose. In this study they assumed that scaling laws for a wall jet over a fixed boundary can be used for a jet over an erodible rough boundary.However, their predictions of the shear stress caused by a submerged jet over a fixed bed do not describe the physical shape compared with the experimental data that was provided by Ghoma(2011). Also, the authorsdo not show the shear stress change from fixed bed to mobile bed. This change and the transport rates couldenable estimation of the shape of the scour hole physically.

The relationship between the rough fixed bed and mobile bed is expected tolink to the shape of the scour hole. In this study, experimental data will be used to develop an analytical model for the development of the shape of the scour hole.

## 2. OBJECTIVE

The main aim of this study is to understand the effect on wall jet flow of a rough, potentially mobile, sediment bed in an open-channel flow and then the effect of the jet on the local scouring pattern on the rough sediment bed. In this paper we investigate the development of an analytical model that can be used to predict how the shape of the scour holechanges with time. The model will describe the use of a different class of statistical equation that is better able to predict the shape of the bed shear stress distribution. This new class of equation is linked with a sediment transport rate function and models local sediment volumetric conservation to predict the shape of an evolving scour hole.

## 3. THE DEVELOPMENT AND APPLICATION OF AN ANALYTICAL MODEL

### 3.1 SHEAR STRESS DISTRIBUTION

The scour problem is caused by the local movement of sediment beneath the turbulent jet. Given that the local sediment transport is strongly related to bed shear stress, a model that is able to simulate the temporal change in shear stress pattern is necessary to predict the unsteady development of the shape of the scour hole. As reported above, Hogg et al. (1997) developed an analytical model to describethe progressive erosion of an initially flat bed of grains by a horizontal turbulent jet. Their study applied a jet flow scaling of streamwise bed shear stress developed over fixed rough beds to mobile beds. For the rough fixed bed they proposed that the streamwise variation in bed shear stress is given by:

$$\tau_b = C_5 \rho\, u_0^2 \left(\frac{x}{b_0}\right)^{-2m+n-1} \left(\frac{b_0}{k_e}\right)^{2(n-2m)} \qquad (3)$$

For a mobile bed they assumed that the bed shear stress for the flow of a two-dimensional jet over an erodible boundary is equivalent to the flow of a two-dimensional jet over a fixed rough boundary is given by equation (3) multiplied by a factor *G(h,x).* They suggest that the streamwise shear stress of an potentially erodible bed is given by:

$$\tau_b = C_6 \rho\, u_0^2 \left(\frac{b_0}{d}\right)^{2\gamma} \left(\frac{x}{b_0}\right)^{-1+\gamma} G\,(h,x) \qquad (4)$$

In this equation they suggest that *G(h,x)* is a shape function to account for the change in the shear stress profile when the boundary is no longer horizontal. The *G(h,x)* function for the fixed bed condition where the boundary is flat ($h$=0) is given by *G(0,x)*=1, but other than this assumption there is no available model or data to predict the variation of G. They proposed a Gaussian G (h,x) function as follows:

$$G\,(h,x) = \begin{cases} 1 & (h \geq 0) \\ \exp\left(-\left(\frac{h}{c_7\ \delta}\right)^2\right) & (h < 0) \end{cases}$$

(5)

Where $C_7$ is a constant and $\delta\,(x)$ is the boundary layer thickness, which varies with downstream position.

Figure (1)shows the non-dimensional bed shear stress variation with distance over a rough fixed bed as calculated from Hogg et al. Equation (4) and the experimental results for streamwsie distribution of Reynolds stress ($-\rho\overline{\boldsymbol{u'w'}}$) obtained in this study for afully fixed rough bed. The experimental data presented in figure (1) shows the mean streamwise profile of Reynolds stress for 11 tests that were reported by Ghoma(2011). All the experiments indicated that the position of the maximum Reynolds stress was not located close to the wall jet. Also the experimental results show that Reynolds stress decreases with an increase in streamwise distance ($x/b_0$) after the location of the maximum Reynolds stress. The equation proposed by Hogg et al. (1997) to describe the shear stress distribution does not describe the shape of the measured shear stress in two key ways. Firstly the zone close to the jet inlet has an increasing pattern of bed shear stress, so that the direction of the gradient compared to the relationship of Hogg et al. is different,and secondly the maximum streamwise gradient is not at x=0, but at x≈15. These two factors will have a significant impact on the erosion rate as this is dependent on the rate of change of the bed shear stress in the streamwise direction.

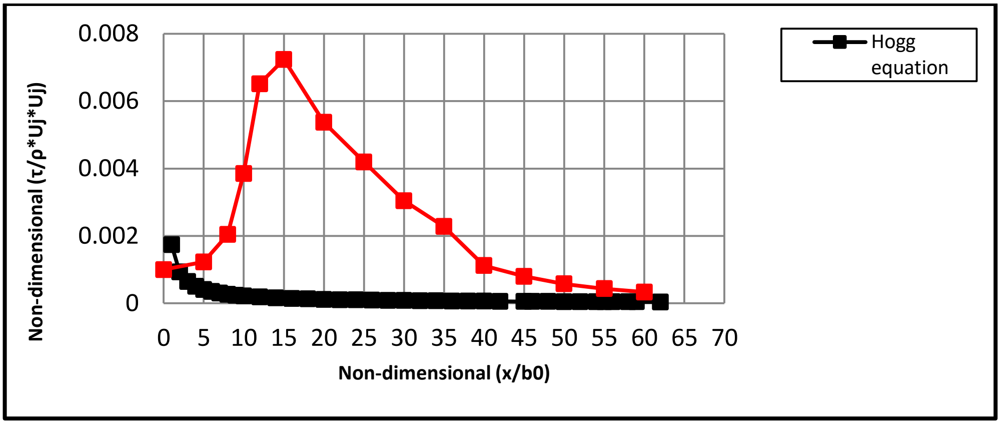



**Figure 1. shear stress distributions over fixed rough bed**

Hogg et al. (1997) have modified the results for flows over fixed boundaries to provide a prediction of shear stress over erodible boundaries. They then balanced the mobilizing and resisting moments on the particles at the surface of the sloping bed, to obtain critical conditions for incipient particle motion with regard to the local bed slope. The calculation of steady-state erosion profiles relied on a simple model of the change in the distribution of shear stress, which neglects regions of flow separation and recirculation.

The results from several tests showed that the streamwise Reynolds stress distribution over a fixed bed commonly had a convex form. The Reynolds stress acting on the equilibrium scoured bed was calculated from the measured streamwise and vertical flow velocity data (Ghoma, 2011). The data collected has been used to first identify new statistical equations thatmay be potentially more suited to describe the streamwise pattern of bed shear stress and then to calibrate these equations so that any modified analytical model of unsteady scour hole development can be tested. It was assumed that the models were most sensitive to shear stress pattern so the method of estimating the local sediment transport rates was not changed. The sediment on the bed surface can be assumed to be potentially susceptible to a motion once the local bed shear stress $(\tau_b)$ exceeds a characteristic critical bed shear stress $(\tau_c)$. The bed shear stress over a rough fixed bed and inside the scour hole is calculated by a new form of relationship as follows:

$$\frac{\tau_b}{\rho * u_j * u_j} = A \sin\left(B + C\left(\frac{x}{b_0}\right)\right)\left(1 - \tanh\left(D\left(\frac{x}{b_0}\right)\right)\right), \quad (6)$$

where A, B, C and D are empirically derived constants. The constants in the above equation of bed shear stress are estimated from the experimental data for different conditions as shown in Table 3.2 (Ghoma, 2011). The performance of this method is examined by calculating the differences between the observed and predicted values of boundary shear stress as shown in Figures (2) and (3).

## 4. EXPERIMENTAL AND ANALYTICAL RESULTS

### 4.1 REYNOLDS STRESS AND EQUATION (6)

Figures (2) and (3) show a comparison between the Reynolds stress obtained using available experimental data and equation (6). The results show that the predicted Reynolds stress fromequation (6)does not match well close to the wall jet, but gives much better results after $x/b_0 \approx 15$.

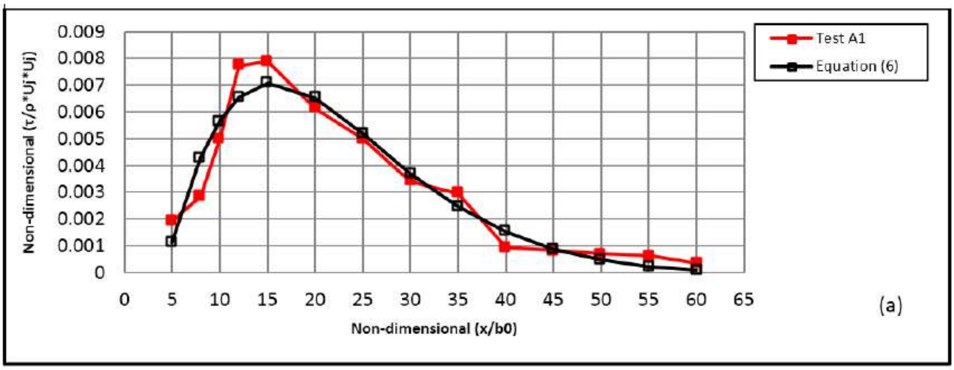



**Figure (2).Comparison of non-dimensional Reynolds stress over rough fixed bed at wall jet (b0 = 0.01m)**

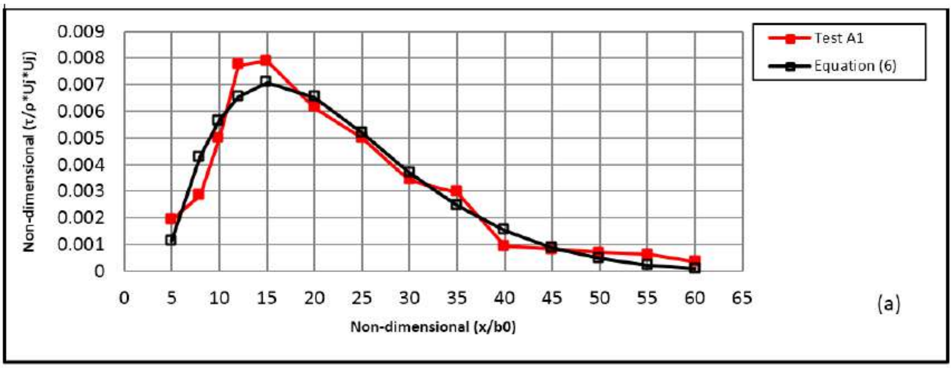



**Figure (3)Comparison of non-dimensional Reynolds stress over rough fixed bed at wall jet (b0 = 0.02m)**

The values of the calibration constants determined using the experimental data are shown in Tables (1) and (2), together with the average size of the error for bed Reynolds stress calculated using equation (6) in comparison to the experimental data. When the wall jet height was (b0 = 0.01m), the constant value for equation (6) is shown in Table (1), and when it was (b0 = 0.02m), it is shown in Table (2). The findings demonstrate that the

Reynolds stress increases as the height of the wall jet increases from 0.01 to 0.02 meters.

**Table 1. Values ofempirical calibration constants for equation (6) for awall jet height $b_0$ = 0.01m**

| Test | Equation(6) | | | | RMS error non-dimensional Reynolds stress |
|---|---|---|---|---|---|
| | A | B | C | D | |
| A1 | -0.0360 | 15.502 | 0.0493 | 0.0477 | 0.0025 |
| A2 | -0.0254 | 15.399 | 0.0663 | 0.0452 | 0.0028 |
| A3 | -0.0215 | 15.407 | 0.0708 | 0.0475 | 0.0020 |
| A4 | -0.0318 | 15.419 | 0.0578 | 0.0531 | 0.0032 |
| A5 | -0.0284 | 15.489 | 0.0454 | 0.0398 | 0.0025 |
| A6 | -0.0323 | 15.487 | 0.0433 | 0.0458 | 0.0026 |

**Table 2.Values for empirical callibration constants for equation 5.6 for awall jet height $b_0$ = 0.02m**

| Test | Equation 6 | | | | RMS error non-dimensional Reynolds stress |
|---|---|---|---|---|---|
| | A | B | C | D | |
| B1 | -0.0444 | 15.291 | 0.1732 | 0.0943 | 0.0049 |
| B2 | -0.0402 | 15.461 | 0.1215 | 0.085 | 0.0019 |
| B3 | -0.156 | 15.623 | 0.0390 | 0.1088 | 0.0035 |
| B4 | -0.0739 | 15.329 | 0.1551 | 0.1089 | 0.0054 |
| B5 | -0.0719 | 15.427 | 0.1213 | 0.1061 | 0.0024 |
| B6 | -0.0328 | 15.352 | 0.1519 | 0.0748 | 0.0035 |

Reynolds stresses calculated using equation 6 and the available experimental data are compared in Figure 4. The findings demonstrate that equation 6 performs poorly in the vicinity of the wall jet inside the scour hole, but performs well in the region after x/b0 15, where the non–dimensional error was 0.0035.

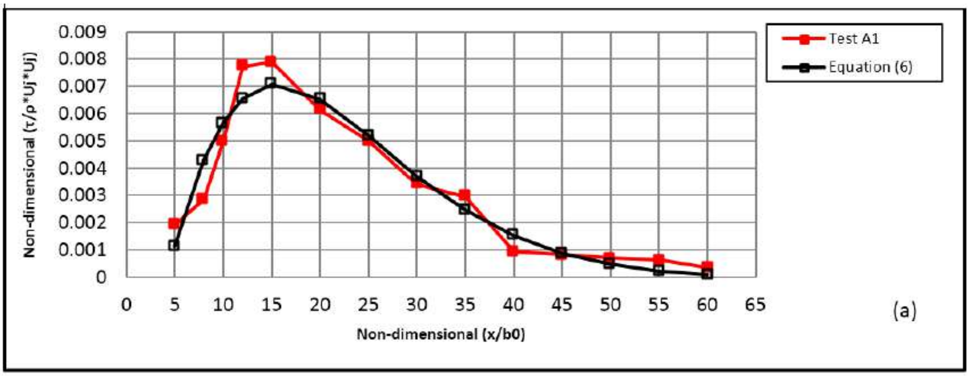



**Figure 4.Comparison of Reynolds stress inside the scour hole**

## 4.2 REYNOLDS STRESS AND EQUATION (8)

The results for the flow over a fixed rough bed and a mobile bed show that the sensitive part to the Reynolds stress is the region close to the wall jet where there is flow recirculation. The results also show that equation 6doesnot work well close to the wall jet but gives good results and fits well with the experimental observations after $x/b_0 \approx 15$.

A new distribution was studied that was believed to address the concerns described above. A distribution was required that was able to simulate the rapid streamwise increase in bed shear stress observed just downstream of the jet entry. It was considered that a distribution with two zones of exponential growth and decay, and a continuous transition was needed. Barndorff-Nielsen (1978) developed a family of probability distributions with exponential power growth and tail-off to describe such a probability function and is in the form:

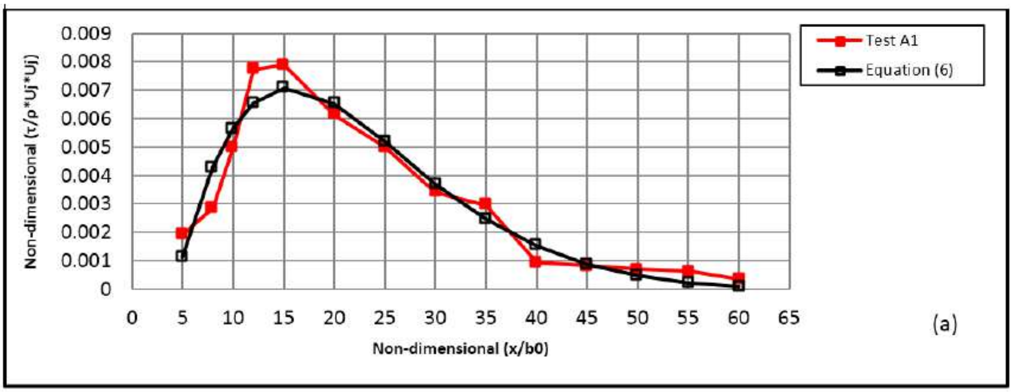
$$P(x;\ \varphi,\ \gamma,\ x,\ \Psi,\ \mu,\ \delta) = a\ (\varphi,\ \gamma,\ x,\ \Psi,\ \delta)\ Exp\ -\frac{1}{2}\left(\varphi\left(\sqrt{\delta^2+(x-\mu)^2}-(x-\mu)\right)^{x}+\gamma\left(\sqrt{\delta^2+\ (x-\mu)^2}+\ x-\mu\right)^{\psi}\right) \quad (7)$$

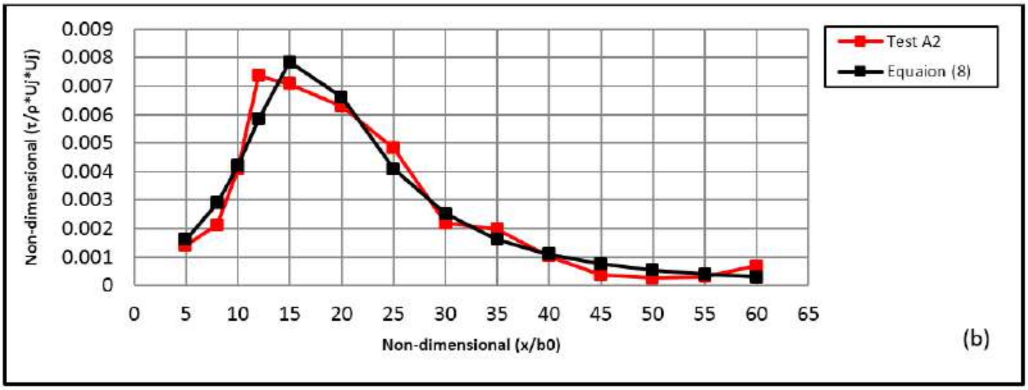
Where a (φ, ɣ, x, Ψ, δ) is the normalising constant, $\mu \in (-\infty, \infty)$ is a location parameter, δ>0 is a scale parameter, and where the remaining four parameters φ, ɣ, x and Ψ are all > 0 and describe the rate of growth and decay of the exponential tails of the distribution. Given the shape of this distribution and its similarity to the experimental data it was decided to use this distribution in this study. The equation for the distributions developed by Barndorff-Nielsen (1978) can be written as:

$$\frac{\tau_b}{\rho * u_j * u_j} = Exp\ -\frac{1}{2}(\varphi(\sqrt{\delta^2 + (x-\mu)^2} - (x-\mu))^x + \gamma(\sqrt{\delta^2 + (x-\mu)^2} + x - \mu)^{\Psi}) \quad (8)$$

Where (φ, ɣ, x, Ψ, δ) are the constants. The constants in the developed equation of bed shear stress are estimated from the experimental data for different conditions. The difference between the observed and predicted values of boundary shear stress is shown in figures 5 and 6.

Figures 5 and 6 show a comparison between Reynolds stress obtained using available experimental data and equation 8. Figure 5 shows the relationship between the experimental data and equation 8 when the wall jet height was $b_0$ = 0.01m,with different velocity inlet.Figure 6 shows the same relationship with the same experimental data but with different wall jet height $b_0$ = 0.02m. The results show that equation 8works well close to the wall jet.


Test A1
Equation (6)
Non-dimensional (τ/ρ*Uj*Uj)
Non-dimensional (x/b0)
(a)


Test A2
Equaion (8)
Non-dimensional (τ/ρ*Uj*Uj)
Non-dimensional (x/b0)
(b)

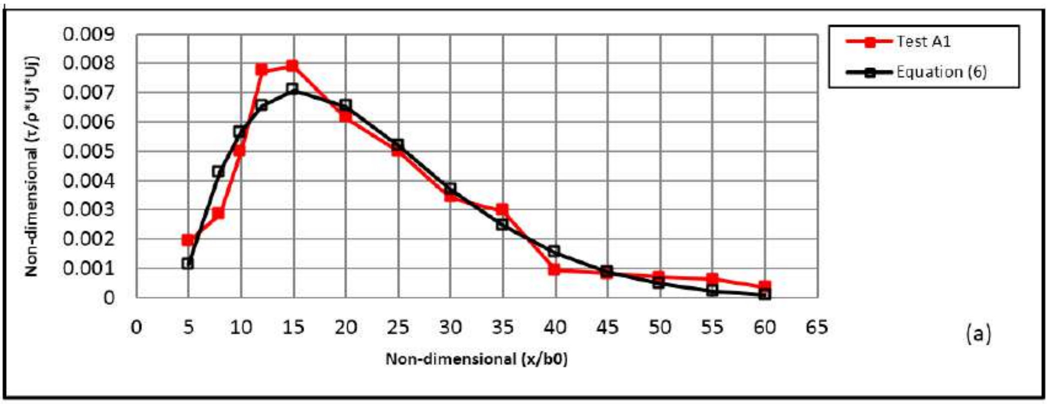
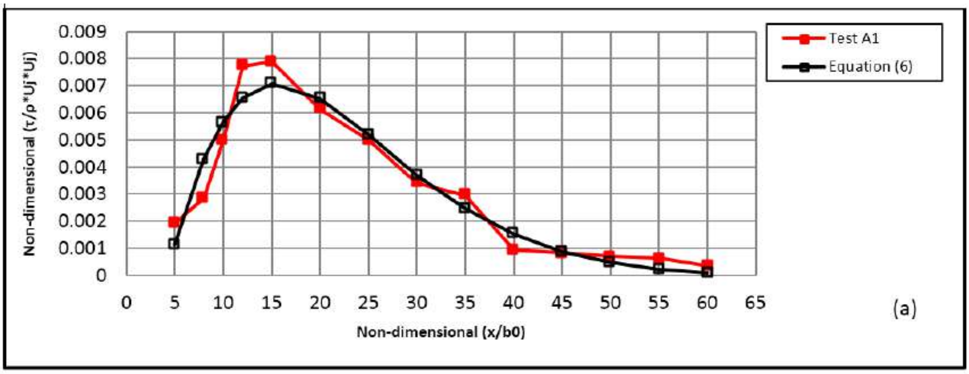
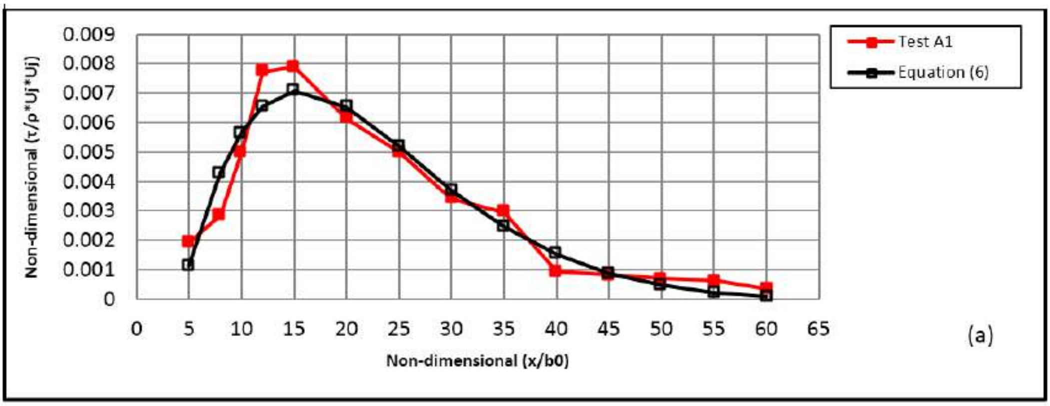
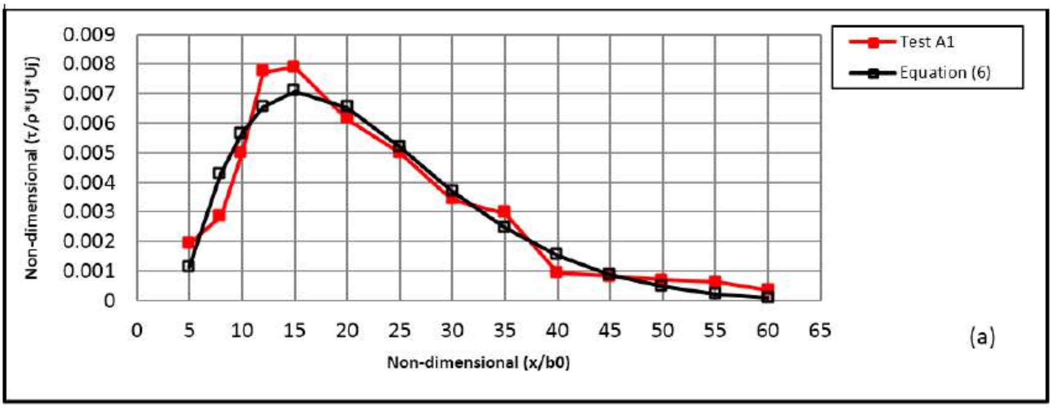
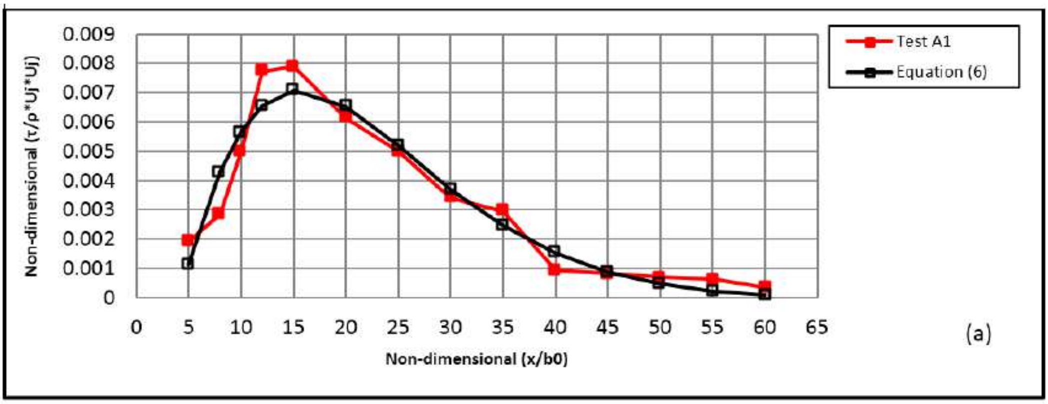


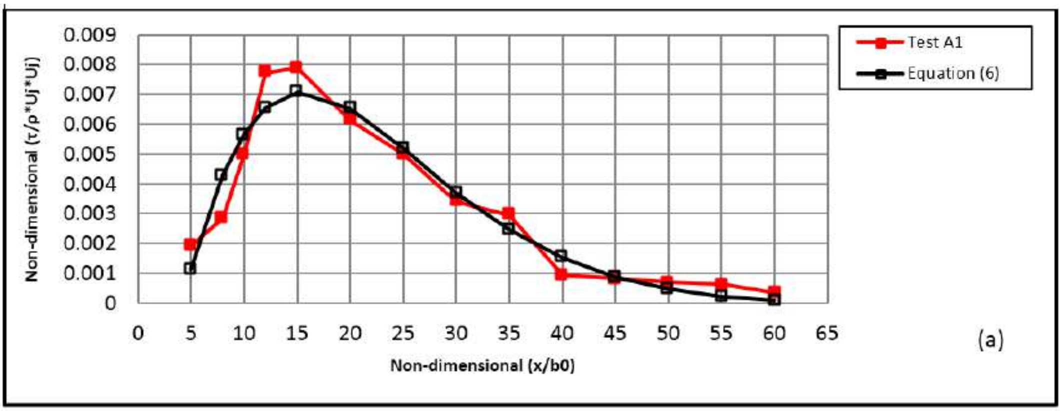

**Figure 5. Comparison of non-dimensional Reynolds stress over rough fixed bed at wall jet (b0 = 0.01m)**




Test A1
Equation (6)
Non-dimensional (τ/ρ*Uj*Uj)
Non-dimensional (x/b0)
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65
(a)


Test A1
Equation (6)
Non-dimensional (τ/ρ*Uj*Uj)
Non-dimensional (x/b0)
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65
(a)


Non-dimensional (τ/ρ*Uj*Uj)
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65
Non-dimensional (x/b0)
Test A1
Equation (6)
(a)


Non-dimensional (τ/ρ*Uj*Uj)
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65
Non-dimensional (x/b0)
Test A1
Equation (6)
(a)

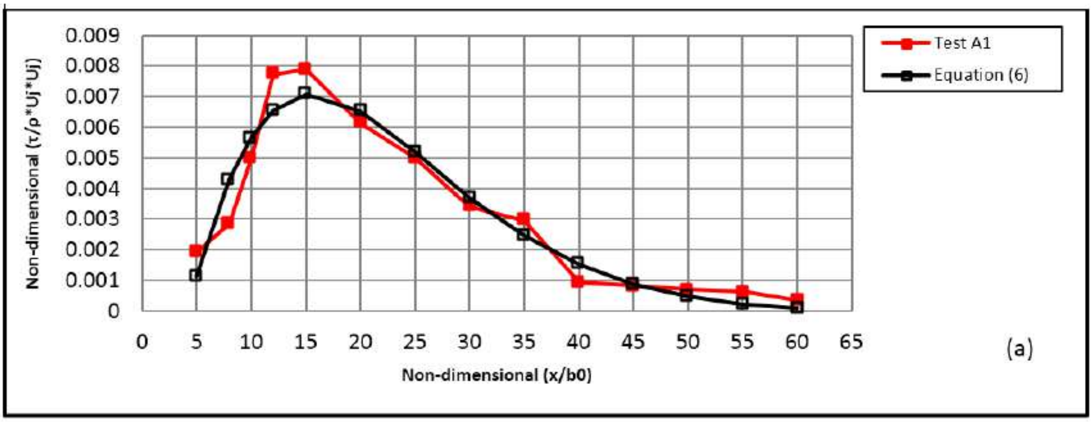



**Figure 6Comparison of non−dimensional Reynolds stress over rough fixed bed at wall jet (b0 = 0.02m)**

When utilizing equation 8 to compare predictions of the bed shear stress distribution with data, the values of the empirical calibration constants and the error for each test are shown in Tables 3 and 4.

Tables 3 and 4 indicate significant improvement for the overall fitting accuracy for Reynolds stress over rough fixed bed. The results show that the parameter $4.32<\delta < 6.73$ is a scale parameter and the parameters φ, γ, x and Ψ are the remaining parameters. Tables 3 and 4 show that the parameters γ and x increased as the velocity increased but the parameter φ decreased and Ψ is almost stable. Thedistribution of these parameters gives a good agreement to the Reynolds stress of the experimental data in all tests. In test A1 the shape factor value increases from 0.002 to 0.008 as we go from left to the right on the horizontal axis as the location parameter μ in equation 8 then starts to decrease towards zero. In general equation 8works well in all regions and fits well with the experimental data.

**Table 3Values of the calibration constants value for equation 8 at wall jet height $b_0$ = 0.01m**

| Test | Equation 8 | | | | | RMS error non-dimensional Reynolds stress |
|---|---|---|---|---|---|---|
| | φ | ɣ | x | ψ | δ | |
| A1 | 1.6767 | 3.1482 | 0.5411 | 0.3373 | 5.2317 | 0.0021 |
| A2 | 1.6268 | 3.001 | 0.5798 | 0.3640 | 5.1930 | 0.0022 |
| A3 | 1.6746 | 3.0170 | 0.5673 | 0.3796 | 5.2355 | 0.00077 |
| A4 | 0.0513 | 7.2488 | 1.6859 | 0.15955 | 4.3278 | 0.0018 |
| A5 | 0.7016 | 5.2845 | 0.8249 | 0.2092 | 4.6896 | 0.0019 |
| A6 | 0.6686 | 5.2823 | 0.8483 | 0.2242 | 4.6760 | 0.0020 |

| Test | Equation 8 | | | | | RMS error non-dimensional Reynolds stress |
|---|---|---|---|---|---|---|
| | φ | ɣ | X | ψ | δ | |
| B1 | 5.599E-04 | 5.1299 | 2.8548 | 0.5574 | 4.6837 | 0.0027 |
| B2 | 6.29E-04 | 6.1009 | 2.7673 | 0.2446 | 6.4972 | 0.0036 |
| B3 | 4.36E-05 | 6.3264 | 3.4813 | 0.2638 | 6.7332 | 0.0048 |
| B4 | 4.97E-06 | 5.0919 | 4.2527 | 0.4617 | 5.9038 | 0.0055 |

| | | | | | | |
|---|---|---|---|---|---|---|
| B5 | 1.02E-06 | 5.8181 | 4.6446 | 0.3075 | 6.7125 | 0.0038 |
| B6 | 1.41E-06 | 5.8207 | 4.6472 | 0.3216 | 6.7140 | 0.0045 |

**Table 4Values of the calibration constants value for equation 8at wall jet height $b_0$ = 0.02m**

### 4.3 EXPERIMENTAL AND EQUATION (8)

Figure 7 shows a comparison between the bed shear stress obtained using experimental data and the theoretical shear stress (using equation 8) inside the scour hole. The agreement is reasonably good with an average error in non-dimensional stress of0.0017.

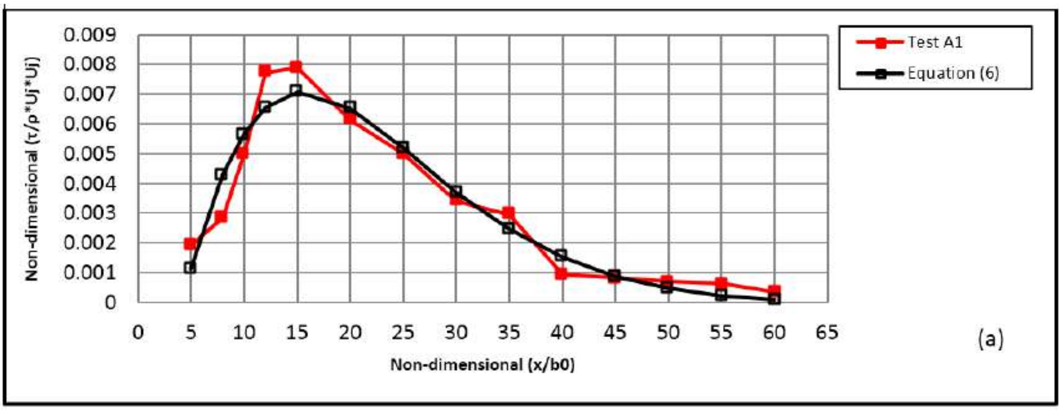
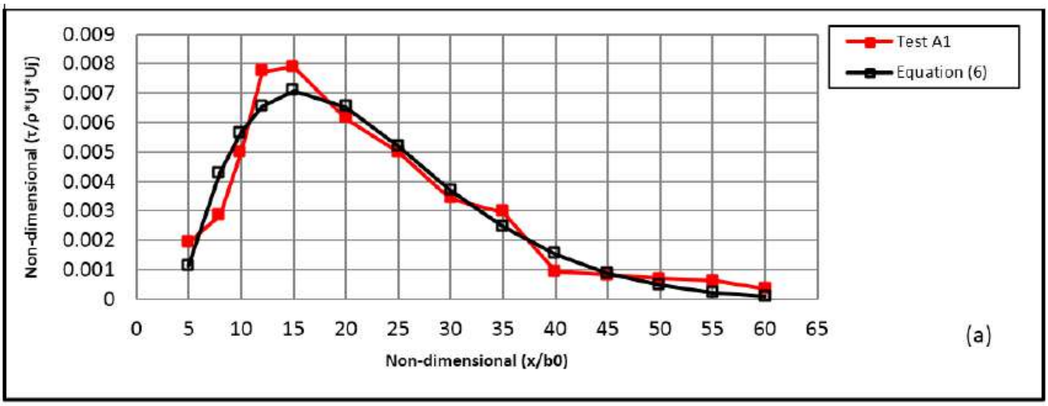



**Figure 7Comparison of Reynolds stress inside the scour hole**

Figure 8 shows the Reynolds stress change from fixed bed to mobile bed by changing the parameter (ɣ) in equation 8. This equation was carried out with the change of that parameter (ɣ) presenting a new way for estimating the shape of the scour hole. The relationship to predict the changing bed shear stress pattern from fixed bed to mobile bed was used to calculate local sediment transport and then the temporal changes in the scour hole shape.



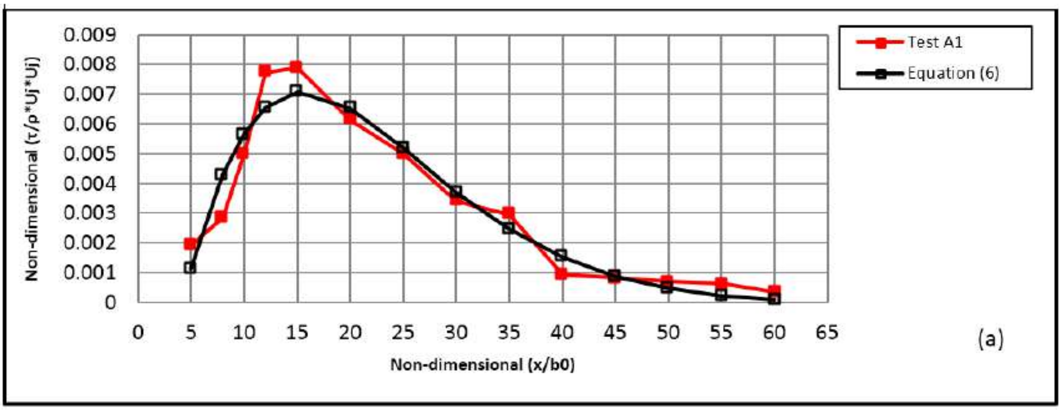




**Figure 8Non-dimensional Reynolds stress changing (a) over a rough fixed bed and (b) inside the scour hole**

Figure 8 demonstrates that, with a very slight error of roughly 0.0017, that the Barndorff-Nielsen equation (Equation 8) closely matches the laboratory results. In contrast to earlier formulae, it also produces good results in the area near the wall jet. Local sediment transport and subsequent temporal variations in the geometry of the scour hole were calculated using the new type of relationship to forecast the changing bed shear stress pattern.

### 4.4 BEDLOAD TRANSPORT

The distribution of bed shear stress throughout the bed surface can be used to compute the contour of the scour hole. The maximum scour depth, scour shape, and scour length are some of the arguments used to model the design parameters of the scour process. The majority of the time, when sediment is transported by a flow as a bed-load, it is in the form of bed material particles that are sliding and rolling directly above the bed. In the literature, there are numerous bed-load transfer rate formulae. Most of them connect a shear stress above a critical bed shear stress value to a bed-load transport rate. This study made use of the Meyer-Peter-Muller formula:

$$q_b = 8\left(\frac{\Delta\rho g d_{50}^3}{\rho}\right)^{1/2}(\tau_b^* - \tau_c^*)^{3/2}, \qquad (9)$$

where $\Delta\rho = \rho_s - \rho$. The critical shear stress parameter, $\tau_c^*$ , with respect to $\tau_{crt}^*$ (for an almost flat bed) depends on the bed slope angle $\beta$ and the angle of repose $\varphi$ in the form of:

$$\tau_c^* = \tau_{crt}^* \frac{\sin(\varphi+\beta)}{sin\varphi} \qquad (10)$$

The functional form of (9) was derived for particle transport on a mobile bed. Then the temporal evolution of the scour hole can be studied by adopting a sediment conservation equation (the Exner equation). In two dimensions, this gives

$$(1-p)\frac{\partial h}{\partial t} + \frac{\partial q_b}{\partial x} = 0 \qquad (11)$$

Where $p$ is the porosity of the bed. The relationship between Reynolds stress and transport rate is then used to predict the changing location of the surface of the scour hole. Mean average Reynolds stress was used in the analytical model with 7 time steps. The duration of the time step was increased as the velocity decreased because of the small transport rates.

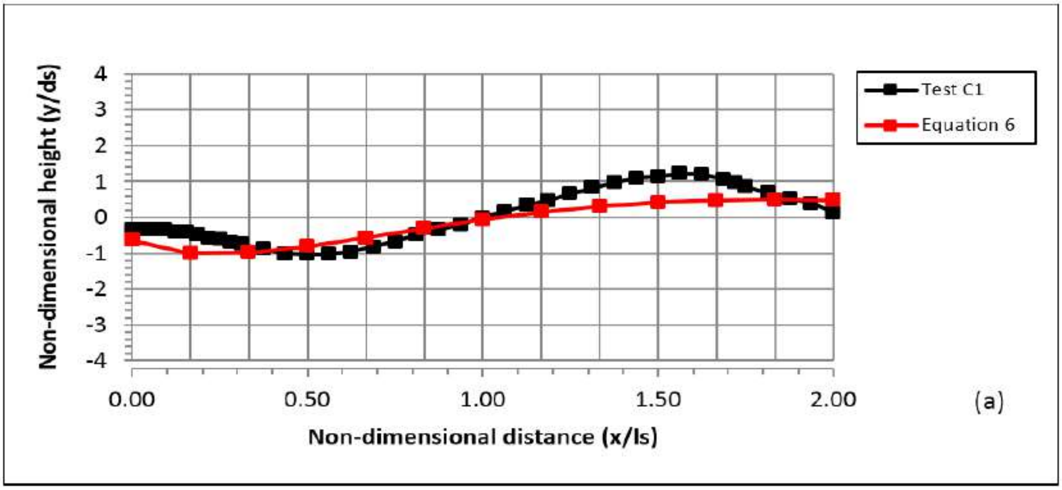
### 4.5USE OF THE MODEL AS A PREDICTION TOOL

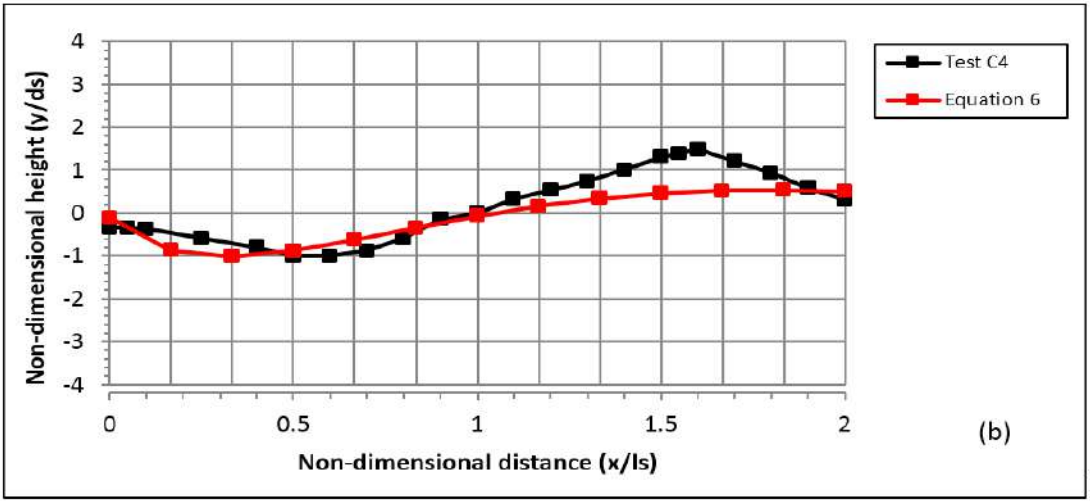
By taking into account the shear stress distribution along the bed's surface, the profile of any scour hole may be computed. Using the models created in this study, three alternative scour profiles were estimated. The following figures show the comparison between the output and the experimental data for the scour hole profile predictions made with MATLAB code.

| Test | Q $[m^3/s]$ | $U_{jet}$ [m/s] | $b_0$ [m] | $F_r$ [-] | $R_e$ [-] | $h_2$ [m] | $h_1$ [m] | $h_w$ [m] | $d_{50}$ [m] | Time of run (hour) |
|---|---|---|---|---|---|---|---|---|---|---|
| C1 | 0.0014 | 0.72 | 0.01 | 2.29 | 7165.54 | 0.165 | 0.108 | 0.08 | 0.00063 | 10 |
| C2 | 0.0013 | 0.66 | 0.01 | 2.11 | 6598.27 | 0.15 | 0.10 | 0.08 | 0.00063 | 10 |
| C3 | 0.001 | 0.50 | 0.01 | 1.59 | 4976.07 | 0.135 | 0.10 | 0.08 | 0.00063 | 10 |
| C4 | 0.0009 | 0.45 | 0.01 | 1.43 | 4478.46 | 0.125 | 0.10 | 0.08 | 0.00063 | 10 |
| C5 | 0.0008 | 0.40 | 0.01 | 1.27 | 3980.85 | 0.120 | 0.10 | 0.08 | 0.00063 | 10 |
| C6 | 0.0006 | 0.35 | 0.01 | 1.11 | 3483.25 | 0.115 | 0.10 | 0.08 | 0.00063 | 10 |

**Table 5. The experimental conditions for tests with mobile bed**

Figure 9 (a, b and c) shows the comparison between the experimental data for Tests C1, C4 and C6 (Table 5) and equation 6 of a scour hole formed by a wall jet. The result showed asimilar depth of scour hole but did not give the right position of the bed material downstream. The results showed poor agreement with experimental data on the evolution of the scour with time.



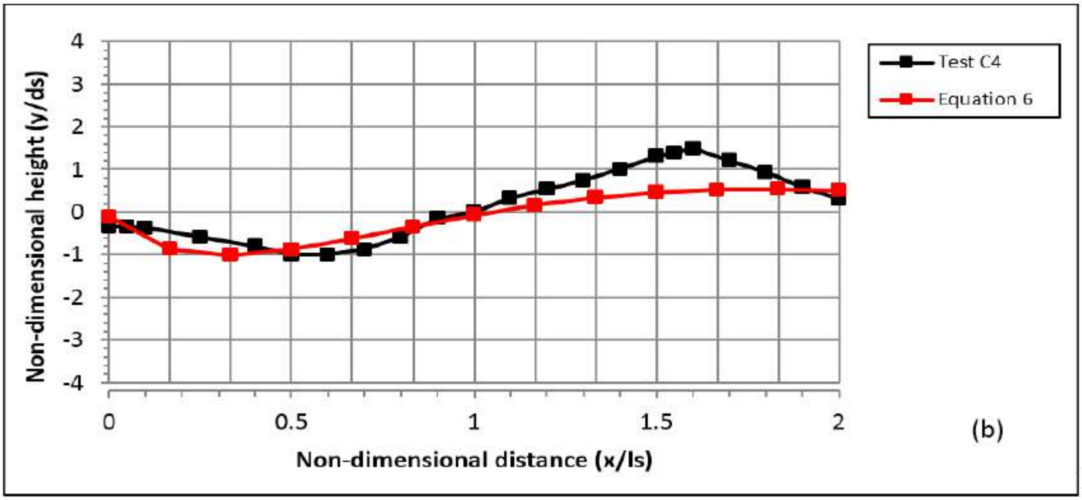
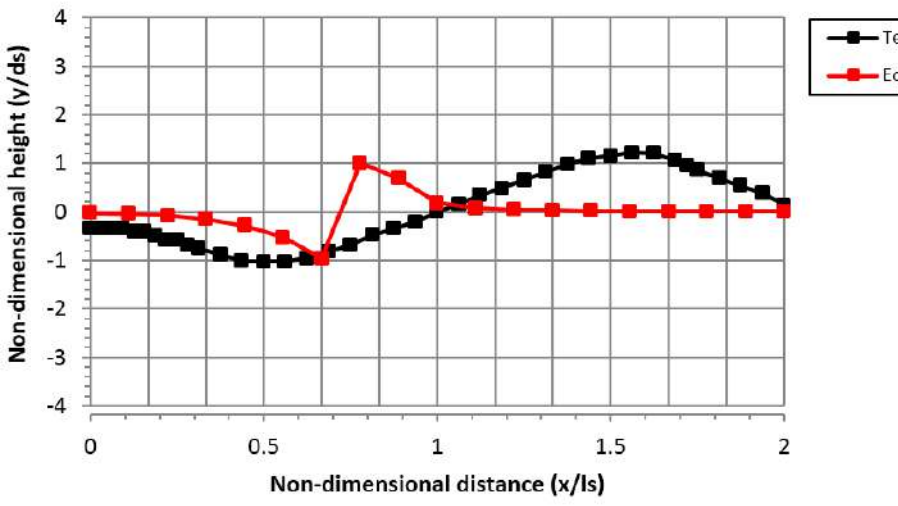
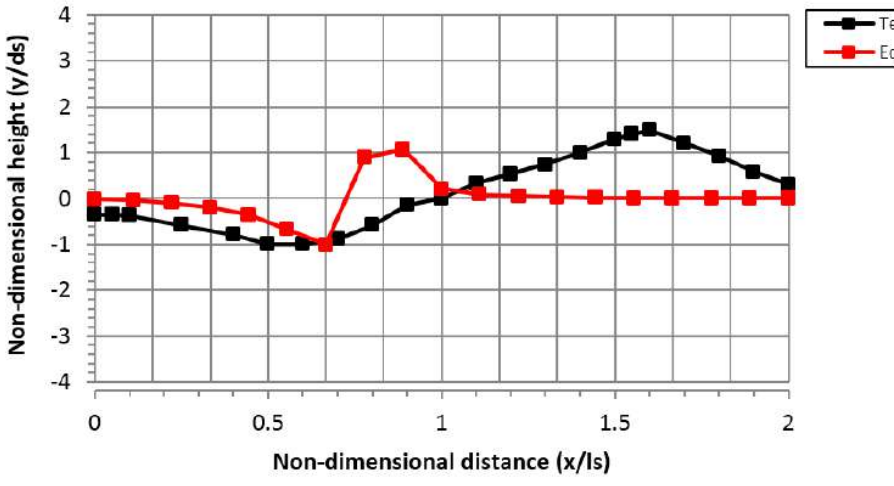



**Figure 9.Comparison of scour holes for Analytical model and Experimental data for (a) Test C1, (b) Test C4 and Test C6**

Figure 10 (a, b and c) shows the comparison between the results of the experimental tests C1, C4 and C6 (Table 5) and the analytical model using equation 8. In this figure, the red colour is the analytical model results and the black colour is the experimental data. It is observed that the analytical model gives the same depth as the laboratory results and also gives the peak position of the sediment dune. The comparisons between the experimental data and the analytical model need more improvement. The analytical model was less successful but it is the first model to give the relationship between Reynolds stress and transport rates to predict the shape of the scour hole.


Test C1
Equation 8
Non-dimensional height (y/ds)
Non-dimensional distance (x/ls)
4
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2

(a)


Test C4
Equation 8
Non-dimensional height (y/ds)
Non-dimensional distance (x/ls)
4
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2

(b)

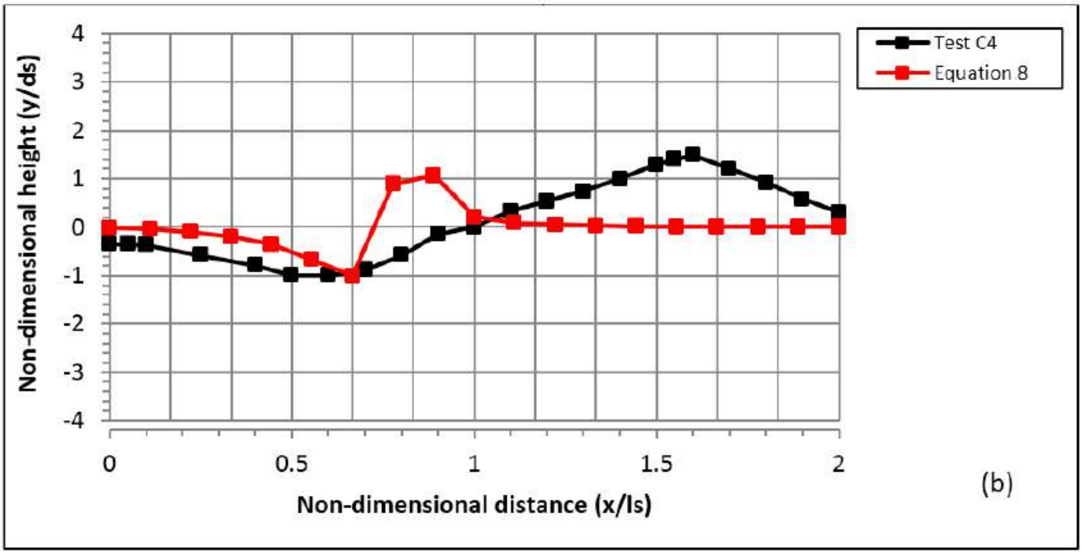



**Figure 10.Comparison of scour holes for Analytical model and Experimental data for (a) Test C1, (b) Test C4 and Test C6**

Table 6 presents the comparison between the experimental data and equations 6 and 8. Three different tests of the scour hole were calculated using equations 6 and 8, developed in this section. For scour hole depth and length, comparisons were between the experimental data and analytical solution. The results indicate that both equations (6 and 8) give similar scour depth but do not give a similar prediction of length. Overall the maximum scour depth is in good agreement with experimental results. The relationship of equation 8 gives better prediction of the shape of the scour hole.

**Table 6. Comparison of scour holes for Analytical model and Experimental data for Test C1, C4 and C6:**

| Test | $U_{jet}$ (m/s) | $b_0$ (m) | Experimental | | Equation 5.6 | | Equation 5.8 | |
|---|---|---|---|---|---|---|---|---|
| | | | $d_s$ (m) | $L_s$ (m) | $d_s$ (m) | $L_s$ (m) | $d_s$ (m) | $L_s$ (m) |
| C1 | 0.72 | 0.01 | 0.073 | 0.32 | 0.073 | 0.12 | 0.073 | 0.14 |
| C4 | 0.45 | 0.01 | 0.043 | 0.20 | 0.044 | 0.12 | 0.043 | 0.13 |
| C6 | 0.35 | 0.01 | 0.037 | 0.18 | 0.34 | 0.12 | 0.037 | 0.14 |

## 5 CONCLUSION

The shape of the scour holes created by turbulent jets was predicted using a variety of models that were presented in this paper. These models used empirically calibrated analytical expressions to describe the streamwise variation of bed shear stress, critical shear stress, and the resulting local sediment transport rate.

The primary cause of the scouring pattern is the bed shear force that the wall jet produces. Analytical formulae calibrated with experimental data were used to estimate the scour formation. The scour profiles acquired from trials were compared with the predictions of the analytical models. It was clear from this comparison that the final shear stress predictor equation provided more accurate scour forecasts than the earlier model. It can be asserted that this model has produced reasonable outcomes in a variety of circumstances.

Future work should be focused on the investigation of scour shape for non-uniform sediment, which has more practical significance.

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