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Statistical Distribution State Estimation Algorithm to Determine Electric Network State Variables

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Abstract This work proposes a three phase statistical distribution network state estimation algorithm formulated on weighted least mean squares method (WLMS) to calculate the network voltage magnitude at each node on the network. The state estimation method used in this work is based on the methods used on transmission network. To overcome the lack of real time measurements which normally exist on 11kV distribution network, this work uses historical data acquired from billing system authority to make the state estimator observable.

Key words: DSE distribution state estimation algorithm, Distribution Network Topology.

1. Introduction,

The primary purpose of an electric distribution system is to meet the customer's demands for energy after receiving the bulk electrical energy from transmission or sub- transmission substation. There are basically two major types of distribution substations: primary substation and customer substation. The primary substation serves as a load center and the customer substation interfaces to the low voltage (LV) network. Customer substation is referred to a distribution room normally provided by the customer. The distribution room can accommodate a number of HV switchgear panel and the transformer to enable LV connection to the customer incoming switchboard. Depending on the geographical location, the distribution network can be in the form of overhead lines or underground cables. Cables are commonly used in urban areas and overhead lines are adopted for rural areas. Distribution networks start from distribution substations to the service entrance of the electricity consumers, including distribution substations, primary feeders, distribution transformers, and secondary systems [1-2]. The existing distribution networks can only serve the requirements and standards of past decades and are not able to meet renewed duties and upcoming challenges. Distribution systems and loads will be subject to dramatic changes over the next 20 to 50 years. To name a few of the changes, we can mention customers' expected services, the reliability level of the system, the characteristic of the new loads, marginal costs, and existing numerous DG generators also future distribution system heading toward central controlling and these need load and distributed generation information.

In the modern energy management System (EMS) State estimation (SE) program processes a set of raw measurement data and provides a real-time load flow solution which is the basis of the advanced functions for system security monitoring and control.

Real time control of the distribution system requires an estimate of the system state. In the past most distribution system was not monitored, therefore, there was no need for SE. To achieve real time monitoring on some existing distribution system are considered to be costly and more capital were needed. In order to reduce these customers cost on real-time monitoring historical consumption data from billing system were used instead. These data are then fed to load flow calculation technique. Under this condition, distribution system load flow program is often used for planning , such as in computing system losses of distribution feeder configuration for system losses reduction, various technique have been proposed to obtain distribution system load flow solution [3-6] . State estimation (SE) program processes a set of raw measurement data and provide real-time load flow solution .The SE was formulated as a equality constrained weighted least squared problem. A distribution system SE that uses a minimum number of remote measurement was presented in [7]. In literature, the expertise and trends related to some of the main aspects of DSE algorithms are analyzed. For example, the authors of [8] review the requirements of state estimation and the differences between state estimation in transmission and distribution systems. Furthermore, in [9], the authors focus on model-based and forecast aided DSSE algorithms, while in [10]. SE is based on the mathematical relations between the system state variable (e,g. bus voltage magnitudes and angles),and the measurement. Various technique have been used to obtain an SE solution, number of studies [11-12] provide surveys on SE algorithms.

. An iterative procedure based on load flow was used to obtain the distribution system SE. These paper introduce distribution state estimation (DSE) to calculate distribution system state..

2. Development of State Estimator Algorithm

The main part of the state estimation technique is the algorithm used to solve weighted least square which derived in the following equation [13]

$$\hat{f} = \sum_{j=1}^{N_m} w_j \hat{e}_j^{2} = \sum_{j=1}^{N_m} \frac{\hat{e}_j^{2}}{\sigma^2_j} = \sum_{j=1}^{N_m} \frac{\left(z_j - f_j(x)\right)^2}{\sigma^2_j} \qquad (1)$$

Where z is the $(m \times 1)$ measurement vector, $f_j(x)$ is the $(m \times 1)$ vector of nonlinear functions, x is the $(2n \times 1)$ true state vector, N_m is the number of measurement, n is the number of buses and $(1/\sigma^2 j)$ represents measurement reciprocal variance.

This equation is to find the best network state variable. The function used in state estimator is nonlinear. Therefore, an iterative method is introduced; the Newton iterative method is used to find state variable value to minimize the least square error. In this research an state estimator algorithm was developed in C language.

Once the network topology and all the different measurement are introduced, the state estimator algorithm will automatically start finding solution. Fig.1 shows flow chart of that algorithm. At the start, from the topology the algorithm will find the network admittance to be used to calculate power flow equations [13] shown below.

$$\vec{S}_{ij} = (V_i^2 G_{ij} - V_i V_j G_{ij}\cos(\theta_i - \theta_j) - jV_i V_j G_{ij}\sin(\theta_i - \theta_j)) - jV^2{}_i B_{ij} + jV_i V_j\cos(\theta_i - \theta_j) - V_i V_j\sin(\theta_i - \theta_j) \quad (2)$$

The equations of represent active and reactive power are that used in the algorithm to form the Jacobean matrix. Also, the Jacobean can include the voltage measurements derivative. The algorithm also builds the $[R]$ matrix which represents the measurement weighting matrix. After all parts of equation 1 were formed, with the initial value the algorithm will start finding solution.

In order to start the iterative process and initial value for each of the state variables is required. 1 p.u voltages is selected and 0.000001 rad for the angle. Using the initial value of $[x]$, $[H]$ and $[a]$ are calculated. Then the value of $[\Delta x]$ is obtained using equation 3.

Adding this value to $[x]$, the state variables are updated:

$$[x] = [x] + [\Delta x] \quad (3)$$

The next iteration is started beginning with $[x]$ the new initial value of the state variables. If Δx is small enough, the algorithm will stop, and if not it will go for another loop until it converge. Sometime this kind of algorithm will never give solution or it can't converge due to that the gain matrix is singular which it's determinant is zero.

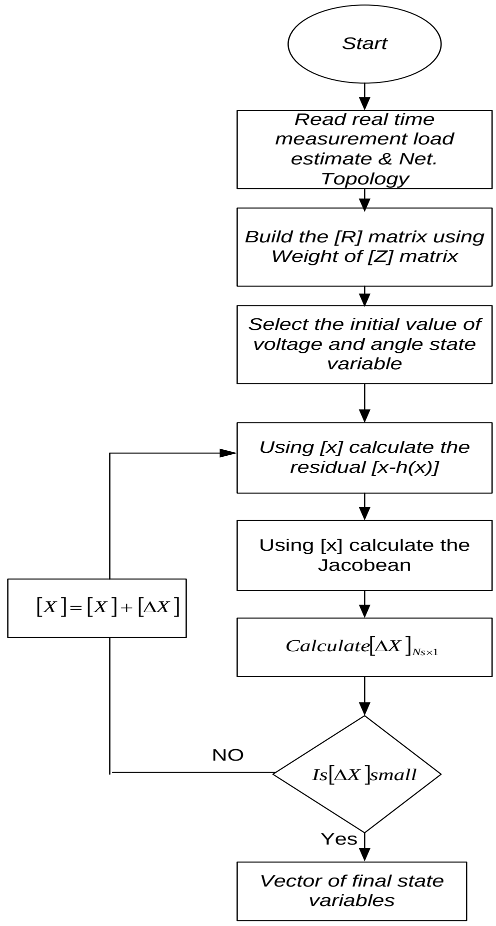

Fig. 1: Flow chart of state estimator algorithm

1.1 State Estimator Requirements

The state estimator required a number of information in order to estimate the system variables or give an output. Figure 3 shows all the possible input information in blocks and the possible output of the state estimator. For more details the state estimator has been divided into an input and output block as follows;

1.1.1 State Estimator Input

The state estimator needs input information in order to give output results. The input information as shown in fig. 2 includes measurements, weighting of these measurement and topology of the network used. The measurements divided into real time measurements, pseudo measurements and virtual measurements.

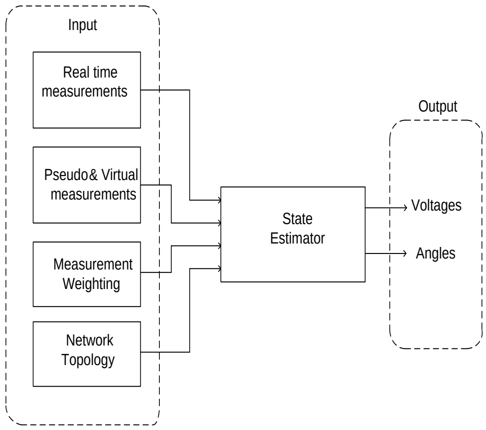

Fig. 2: State estimator blocks

I) Real Time Measurement

The real time measurements are very essential to the state estimator in order to predict system solution similar that of load flow solution. The real time measurements can be voltage magnitude, power flow or current measurement. These measurements are very important to state estimator algorithm. Usually, these measurements are expected to be accurate or having less error. In distribution network because of expenditure control to make the system competitive using for example voltage meter with high accuracy [15] its price increases dramatically. It is found that maximum accuracy of voltage measurement at reasonable price is around $\pm 0.5\%$. The accuracy of an 11 kV voltage measurement should be between $\pm 1$and$\pm 2\%$. In these research to study the influence of distributed generator measuring the active and reactive power were highly needed to achieve good state estimator results so they given high accuracy same as that of real time. Acquiring the DG power is a little bit more difficult to obtain due to location of DGs in distribution system, so remote terminal unit connected through communication is needed. The general practice assumed the accuracy of the DGs power meters should be around $\pm 3\%$.

II) Pseudo Measurements

The pseudo measurements are historic data used to replace the shortage of real time measurements which are required by state estimator. The pseudo measurement can be extracted, for example, through monthly billing data, monthly peak loading, the transformer's peak load analysis and existing diversified load curves. Because this data has big probability of lack of accuracy so it is given low degree of accuracy than the real time measurements.

III) Virtual Measurements

Some of network buses have no load connected to them, therefore the value of active and reactive power (P,Q) consumption in those buses is zero. These virtual measurements act like perfect measurement and very valuable for the distribution state estimator. It assumed extremely perfect. As the weighting in the state estimator used the reciprocal variance and if the error is introduced zero that will give numerical problem to the state estimator, it will collapse. So, it assumed the variance is very small but not zero.

IV) Weighting of Measurements

In this research state estimator all the measurements, pseudo measurements and virtual measurements are assumed to follow normal distribution. Due to the limited availability of real-time data, load demand estimates are used as pseudo measurements. This in conjunction with the weighted-least squares formulation implies that load demands are normally distributed. The normal or Gaussian distribution function is determined by the mean of the data and the standard deviation of the data. If the data has small standard deviation it is more accurate. Fig.3 shows the function of normal distribution function where 99.73% of the curve area was shadowed. The 99.73% is representing that the data has high level of confidence which means that the deviation between the boundary of the interval and the mean value is three times the standard deviation. From the normal distribution function and the standard deviation measurement accuracy can be found using equation 2 as follow:

If the accuracy of a measurement is for example $\pm 4\%$, it can be assumed that the 99.73% of the time the measurement will be inside those given accuracy limits. This assumption allows writing the standard deviation of the distribution using the following equation:

$$3 \cdot \sigma = mean \cdot \frac{accuracy}{100}$$

Exponential

$$\sigma = mean \frac{accuracy}{300} \qquad (4)$$

Where σ is the standard deviation and the mean of the measurement or the estimated load of the pseudo-measurement.

For the given example of a measurement of $\pm 4\%$ accuracy and if the measured magnitude is 0.95 p.u. the standard deviation will be:

$$\sigma = 0.95 \cdot \frac{4}{300} = 0.01266 p.u$$

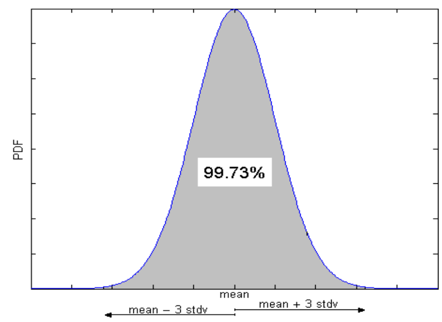

Fig.3: Function of normal distribution [13]

IIV) Network Topology

It is important to define the network for the state estimator in order to calculate line flow and the power consumption at each busbar. The network topology can be represented in the state estimator by line impedance, number of buses and number of branches and the sending end and the receiving end of the branches. Also, the load connected to each busbar.

1.1.2 State Estimation Jacobean

In power system each node has its related differential equation need to be determined to build the system Jacobean [13-16]. The Jacobean is the important part in solving load flow and state estimation problem. It is the matrix forming all differential equation of measurement functions such as active and reactive power measurements of all nodes in relation of state variables which normally means both bus voltage and angles. In order to estimate both voltage and angle magnitudes, the angle at one of the N buses of the system should be taken as a reference for all other angles, which leaves $N-1$ angles and N magnitudes to be calculated by the following equation.

$$x^{(k+1)} - x^{(k)} = (H_x^T R^{-1} H_x)^{-1} H_x^T R^{-1} \begin{bmatrix} z_1 - h_1(x_1^{(k)}, x_2^{(k)}, \ldots, x_{N_S}^{(k)}) \\ z_2 - h_2(x_1^{(k)}, x_2^{(k)}, \ldots, x_{N_S}^{(k)}) \\ . \\ . \\ . \\ z_m - h_{N_m}(x_1^{(k)}, x_2^{(k)}, \ldots, x_{N_S}^{(k)}) \end{bmatrix}$$

It is unlike the square Jacobean of Newton Raphson in load flow, it is always has $(2N-1)$ columns and a larger number N_m of rows. Each row of H_x corresponds uniquely to one of the measured quantities indicated in the network. The general form of H_x matrix is:

$$\begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \frac{\partial f_1(x)}{\partial x_3} & \frac{\partial f_1(x)}{\partial x_s} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \frac{\partial f_2(x)}{\partial x_3} & \frac{\partial f_2(x)}{\partial x_s} \\ \frac{\partial f_m(x)}{\partial x_1} & \frac{\partial f_m(x)}{\partial x_2} & \frac{\partial f_m(x)}{\partial x_3} & \frac{\partial f_m(x)}{\partial x_s} \end{bmatrix}_{N_m \times N_s}$$

the Jacobean matrix is called information matrix and each element in the matrix $H(x)$ can be expressed through mathematical derivation by taking the differential of the equation for each measurement with respect to state variables, bus voltage angle δ and magnitude V. The matrix $H(x)$ is arranged so that each row represents the derivative of the active power with respect to a particular busbar voltage and angle. First column of that row is representing derivative of active power in relation of voltage and second column is representing active power derivative in relation with angle and so on for all busbars. The same was repeated in second row where the derivative of reactive power with respect to voltage and angle is taking place. This continues to all busbars where each busbar should have two rows in the Jacobean.

If the voltage measurement is introduced in the state estimation the last rows will include the derivative value of the voltage at that busbar, see the C program in appendix (A).

The transpose of $H(x)$ is $H^T(x)$ and is equal to:

$$H^T = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_3(x)}{\partial x_1} & \frac{\partial f_m(x)}{\partial x_1} \\ \frac{\partial f_1(x)}{\partial x_2} & \frac{\partial f_2(x)}{\partial x_2} & \frac{\partial f_3(x)}{\partial x_2} & \frac{\partial f_m(x)}{\partial x_2} \\ \frac{\partial f_1(x)}{\partial x_s} & \frac{\partial f_2(x)}{\partial x_s} & \frac{\partial f_3(x)}{\partial x_s} & \frac{\partial f_m(x)}{\partial x_s} \end{bmatrix}_{N_m \times N_s}$$

1.1.3 DSE Validation and Sensitivity Studies on Part of Morton Park

The validation and sensitivity studies were carried out on part of Morton park network shown in fig.5 This network is an 11 kV network with 60 buses distributed into six feeders. Appendix A and B presents line, bus parameters and load consumption.This network has 120 power measurements; same data accuracy which was used in previous case of generic network. The real voltage measurements were connected at slack bus and bus number 47 where the DG is connected.

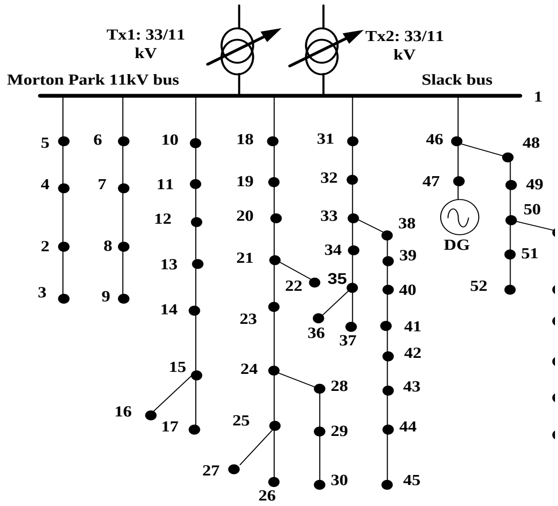

Fig.5: Single line diagram of A 60 nodes of Morton Park network [17]

Fig.6 shows the result of the voltage calculated by the developed C code to each node of the part of Morton Park network

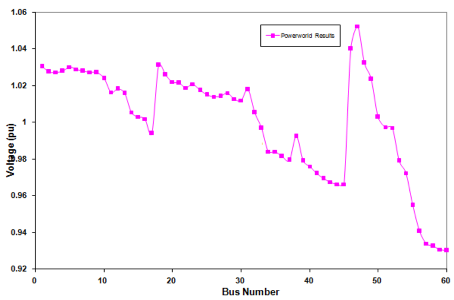

Fig.6: Part of Morton Park node voltage

- DSE Validation study on Part of Morton Park

The result of the state estimation using the developed C code shows similar result to Powerworld load flow package. The state estimation code converges to a result close to the load flow result which was carried out by the Powerworld simulation package. Fig. 7 and fig. 8 show the state estimation results against the Power world result with and without Distributed generator.

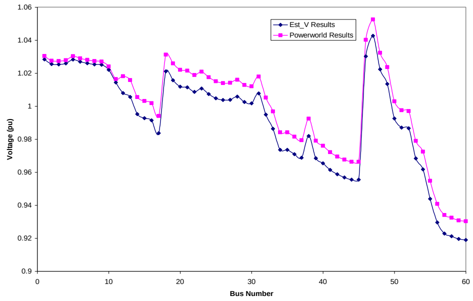

Fig.7: Estimated bus voltage Powerworld results without DG

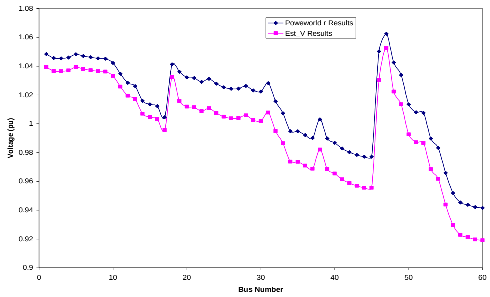

Fig.8: Estimated bus voltage Powerworld results with DG

- Morton Park Network Current and Power Flow Estimation

The developed algorithm also can be used to calculate the Estimated flow of both active and reactive power and flow of current at all branches of the network as shown the following figures respectively.

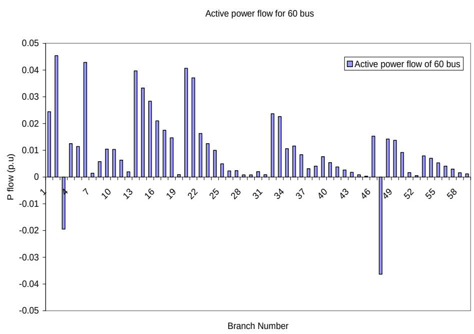

Fig.9: Estimated active power flow of 60 bus network

Fig.9 shows active power of Morton branches. This figure shows that most of power flow has positive direction accept the branch influenced by DG they have negative direction and this indicates power injection from DG side.

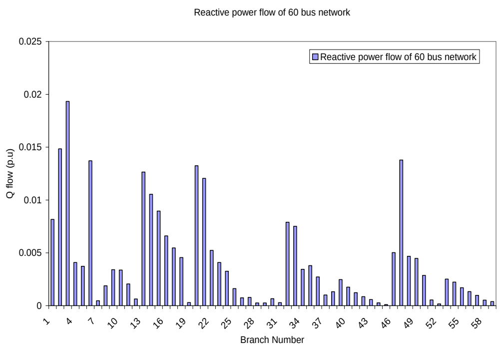

Fig.10: Reactive power flow of 60 bus network

Fig.10 show that directions of reactive power are all positive which means all load and generation are consuming reactive power.

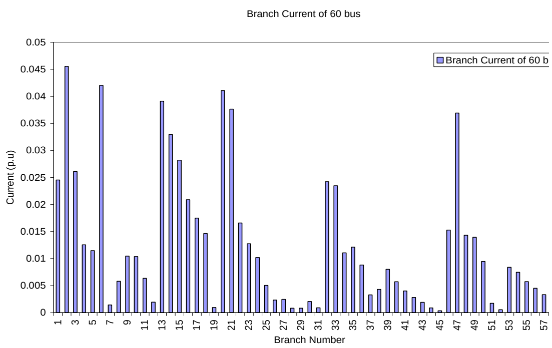

Fig.11 Estimated branch current of 60 bus network

Fig.11 shows the magnitude of branch current of the Morton Park network with one DG connected at bus 47.

1.3 DSE sensitivity study on Morton Park Network

The results presented in this section investigated several parameters affecting the voltage estimation. These parameters are;

1-Number of voltage measurements

2-Accuracy of the load estimate

- Influence of number of voltage measurements on DSE

In this study all the pseudo measurements of 60 were given the accuracy of $\pm 25\%$ and virtual measurements were given very high accuracy of $\pm 1\times 10^{-11}$ and the voltage real time measurements were given accuracy of $\pm 1\%$. In this study the number of real time measurements will increased and all other measurements will stay with fixed accuracy. The state estimation results presented in fig.12 show that bus voltage's uncertainty has increased as the number of real time measurements increasing.

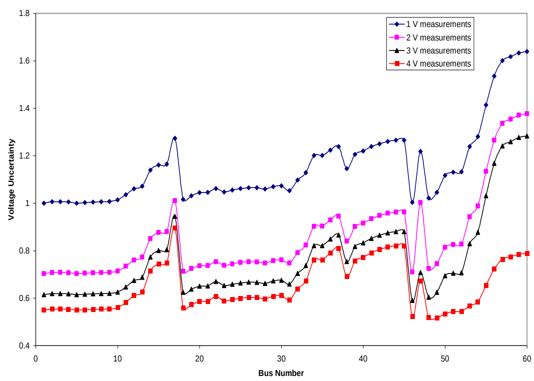

Fig.12: Influence of number real measurement on uncertainty

- Accuracy of the Pseudo Measurements

In this study busbar voltage uncertainty has been checked when all pseudo measurements were given different accuracy (± 25, $\pm 50\%$, $\pm 75\%$, $\pm 100\%$, $\pm 125\%$, $\pm 150\%$, $\pm 175\%$, $\pm 200\%$). The state estimation results presented in fig.13 show that busbar voltage uncertainty has decreased when pseudo measurements accuracy has decreased.

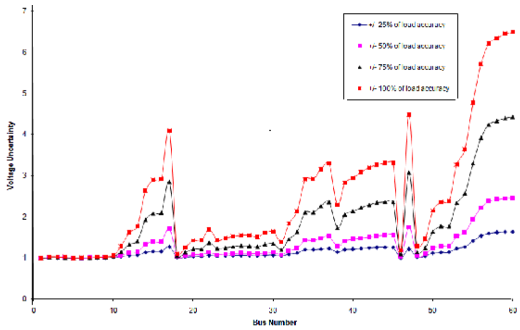

Fig.13: Influence of load accuracy on voltage uncertainty

4 Conclusions

This work presented a three-phase algorithm to determine electric network variables for a distribution system which based on DSE technique. It uses active and reactive power acquired from billing system which used as pseudo measurement to obtain an estimation of real-time system states. Results for the proposed distribution system SE is discussed and there validation was compared and almost show full agreement. The study has carried out the influence of changing the accuracy by giving some high accuracy to some location to reassemble real-time measurement. Also the developed algorithm can calculate line current and a flow of power through all branches of the network which provide away for monitoring and protection. The result gained from the algorithm will facilitate heading toward complete automation and monitoring to any distribution network without causing any change to it's infra structure.

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Appendix A

Morton Park Lines and Buses Parameters

Branch No.	Branch From	To	L. Resistance (pu)	L. Inductance (pu)	L. Cap. (pu)
1	1	31	0.7782	0.2684	0
2	1	18	0.141	0.0724	0
3	1	46	0.1435	0.0476	0
4	1	6	0.1007	0.0439	0
5	1	5	0.0085	0.0035	0
6	1	10	0.1349	0.0627	0
7	2	3	0.1209	0.0278	0
8	4	2	0.0616	0.0268	0
9	5	4	0.2016	0.0992	0
10	6	7	0.0767	0.045	0
11	7	8	0.1048	0.0612	0
12	8	9	0.108	0.0222	0
13	10	11	0.1763	0.0784	0
14	11	12	0.1811	0.0628	0
15	12	13	0.0785	0.0272	0
16	13	14	0.47	0.1285	0
17	14	15	0.132	0.0328	0
18	15	17	0.6037	0.1282	0
19	15	16	1.4219	0.1161	0
20	18	19	0.1185	0.0486	0
21	19	20	0.0902	0.0496	0
22	20	22	0.1695	0.0958	0
23	20	21	0.0314	0.0268	0
24	21	23	0.0595	0.0137	0
25	23	24	0.6292	0.2022	0
26	24	28	0.6229	0.2135	0

27	24	25	0.9646	0.3309	0
28	25	27	1.0514	0.3605	0
29	25	26	1.2303	0.4219	0
30	28	29	1.4529	0.4935	0
31	29	30	0.8663	0.2921	0
32	31	32	0.4676	0.2446	0
33	32	33	0.2669	0.3231	0
34	33	38	0.3605	0.1484	0
35	33	34	0.9755	0.3346	0
36	34	35	0.0035	0.0014	0
37	35	37	1.4758	0.1272	0
38	35	36	0.6213	0.0621	0
39	38	39	1.6864	0.1678	0
40	39	40	0.5393	0.0454	0
41	40	41	0.9985	0.0873	0
42	41	42	0.9483	0.0794	0
43	42	43	0.9978	0.0834	0
44	43	44	1.3631	0.5068	0
45	44	45	0.0934	0.032	0
46	46	48	0.4783	0.1618	0
47	46	47	0.737	1.0936	0
48	48	49	0.5125	0.3818	0
49	49	50	1.2433	0.9109	0
50	50	53	2.563	0.211	0
51	50	51	3.092	0.6508	0
52	51	52	0.8221	0.2819	0
53	53	54	0.7919	0.0665	0
54	54	55	2.3917	0.1964	0
55	55	56	2.4871	0.2063	0
56	56	57	1.5092	0.1241	0
57	57	58	0.4802	0.0392	0
58	58	59	0.9473	0.0774	0
59	59	60	0.4144	0.1347	0

Appendix B

Morton Park Load Consumption and Generation

Bus No.	Load Active Power (MW)	Load Reactive Power (kVar)
1	-11.12016	-4.88432
2	0.43478	0.14291
3	0.14165	0.04656
4	0.46433	0.15262
5	0.09953	0.03271
6	0.21832	0.07176
7	0.40043	0.13162
8	0.43801	0.14397
9	0.19214	0.06315
10	0.28978	0.09525
11	0.60676	0.19943
12	0.46628	0.15326
13	0.7155	0.23517
14	0.3294	0.10827
15	0.18533	0.06091
16	0.0895	0.02942
17	1.40804	0.4628
18	0.44352	0.14578
19	0.33995	0.11174
20	0.82021	0.26959
21	0.25133	0.08261
22	1.6385	0.53855
23	0.50512	0.16602
24	0.02751	0.00904
25	0.07904	0.02598
26	0.07942	0.0261
27	0.07891	0.02594
28	0.02571	0.00845
29	0.11386	0.03742
30	0.08571	0.02817
31	0.02955	0.00971
32	0.07283	0.02394
33	0.02847	0.00936
34	0.31349	0.10304
35	0.11706	0.03848

36	0.40573	0.13336
37	0.30954	0.10174
38	0.29158	0.09584
39	0.21561	0.07087
40	0.15984	0.05254
41	0.11306	0.03716
42	0.08422	0.02768
43	0.09478	0.03115
44	0.04968	0.01633
45	0.03135	0.0103
46	0.15097	0.04962
47	-3.47	1.34
48	0.09423	0.03097
49	0.03641	0.01197
50	0.26742	0.0879
51	0.11449	0.03763
52	0.05039	0.01656
53	0.10096	0.03318
54	0.08687	0.02855
55	0.15872	0.05217
56	0.11064	0.03637
57	0.10723	0.03525
58	0.13834	0.04547
59	0.04198	0.0138
60	0.11615	0.03818