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Optical-Solitons in Magneto-optic Waveguides with Perturbed Resonant Nonlinear-Schrödinger Equation Having Parabolic-Law Nonlinearity and Multiplicative White Noise

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Abstract:

This article focuses on a coupled system of perturbed resonant nonlinear-Schrödinger equation (NLSE) in magneto-optic waveguides. The system incorporates spatio-temporal dispersion (STD), inter-modal dispersion (IMD), parabolic-law nonlinearity, and multiplicative noise in the Itô sense. By using the improve direct algebraic approach and computer algebraic system such as Maple, various types of optical solitons are explored, including dark solitons, bright solitons, straddled solitons, singular solitons, as well as solutions based on Jacobi elliptic functions and Weierstrass-elliptic functions. Numerical simulations of some solutions are also presented.

Keywords: Dispersive optical solitons; Resonant nonlinear-Schrödinger equation; Magneto-optic waveguides; Multiplicative white noise.

1. Introduction

Nonlinear partial differential equations (PDEs) play a crucial role in modeling a wide range of phenomena across the physical sciences, from nonlinear optics to fluid dynamics and plasma physics. In recent decades, researchers have dedicated significant effort to uncovering explicit soliton solutions for these nonlinear systems using diverse mathematical techniques [1-19].

Chief among these models is the nonlinear Schrödinger equation (NLSE), which has emerged as a cornerstone framework for describing the propagation of optical solitons and other nonlinear wave behaviors. While the classic NLSE has been extensively studied, there remains keen interest in exploring extensions and generalizations of this fundamental equation [1-19].

One important consideration is the need to incorporate higher-order effects, such as spatio-temporal dispersion (STD) and inter-modal dispersion (IMD), to more accurately model the propagation of optical solitons in nonlinear media [16-18]. Additionally, when examining phenomena like chiral solitons in the quantum Hall effect, the inclusion of specific resonant terms in the governing equation becomes critical [10-18].

Building upon this foundation, the present work introduces a novel coupled system of perturbed resonant NLSE tailored for magneto-optic waveguides. This system incorporates parabolic law nonlinearity, STD, IMD, and multiplicative noise in the Itô sense. The details and findings of this investigation are presented in the subsequent sections.

2. Governing model

The dimensionless form of perturbed resonant NLSE in polarization preserving fibers with dual power law nonlinearity and both STD, IMD having multiplicative white noise in the Itô sense is written as [16]:

$$i\varphi_t + a\varphi_{xx} + b\varphi_{xt} + (c|\varphi|^2 + d|\varphi|^4)\varphi + \gamma\left(\frac{|\varphi|_{xx}}{|\varphi|}\right)\varphi + \sigma(\varphi - ib\varphi_x)\frac{dW(t)}{dt} = i\delta\varphi_x, \quad (2.1)$$

where $\varphi(x,t)$ is a complex-valued function that represents the wave profile, a, b, c, d, γ, δ and σ are real-valued constants with $i = \sqrt{-1}$. The first term in equation (2.1) is the linear temporal evolution, the constants a and b are the coefficients of chromatic dispersion (CD) and STD respectively. Next c and d are the coefficient of self-phase modulation (SPM), The constant γ is the coefficient of resonant nonlinearity. The constant δ is coefficient of IMD. Finally, σ is the coefficient of noise strength and $W(t)$ is the standard Wiener process, such that $dW(t)/dt$ is the white noise.
In birefringent fibers, equation (2.1) splits into two components, for the first time, as:

$$iu_t + a_1u_{xx} + b_1u_{xt} + (c_1|u|^2 + d_1|v|^2)u + (e_1|u|^4 + f_1|u|^2|v|^2 + g_1|v|^4)u + +h_1\left(\frac{|u|_{xx}}{|u|}\right)u + \sigma(u - ib_1u_x)\frac{dW(t)}{dt} = Q_1v + i[\lambda_1u_x + \mu_1(|u|^2u)_x + \theta_1(|u|^2)_xu + r_1|u|^2u_x], \quad (2.2)$$

And

$$iv_t + a_2v_{xx} + b_2v_{xt} + (c_2|v|^2 + d_2|u|^2)v + (e_2|v|^4 + f_2|v|^2|u|^2 + g_2|u|^4)v + h_2\left(\frac{|v|_{xx}}{|v|}\right)v + \sigma(v - ib_2v_x)\frac{dW(t)}{dt} = Q_2u + i[\lambda_2v_x + \mu_2(|v|^2v)_x + \theta_2(|v|^2)_xv + r_2|v|^2v_x], \quad (2.3)$$

where $u(x,t)$ and $v(x,t)$ are complex-valued functions that represent the wave profiles. The constants a_j and b_j $(j = 1,2)$ are the coefficients of CD and STD in the directions of x and y respectively. The constants c_j and d_j, $(j = 1,2)$ are the coefficients of SPM and cross-phase modulation (XPM) respectively. The constants e_j, f_j and g_j, $(j = 1,2)$ are the coefficients of nonlinear dispersion terms. The constants h_j, $(j = 1,2)$ are the coefficients of resonant nonlinearity. The constants σ_j, $(j = 1,2)$ are the coefficients of noises strength and $W(t)$ is the standard Wiener processes, such that $dW(t)/dt$ is the white noises. Finally, the constants λ_j, μ_j, θ_j and v_j, $(j = 1,2)$ are the coefficients of the IMD, self-steepening (SS) terms and nonlinear dispersions terms respectively.
This article aims to deduce the soliton solutions for equations (2.2) and (2.3) using the enhanced direct algebraic method.

3. Wave Transformation and Mathematical Analysis

In order to solve equations (2.2) and (2.3), we suppose that the wave profiles have the following forms:

$$u(x,t) = \varphi_1(\xi)\,exp[i(\psi(x,t) + \sigma W(t) - \sigma^2 t)], \quad (3.1)$$

$$v(x,t) = \varphi_2(\xi)\,exp[i(\psi(x,t) + \sigma W(t) - \sigma^2 t)], \quad (3.2)$$

$$\xi = x - \rho t, \psi(x,t) = -\kappa x + \omega t, \quad (3.3)$$

where κ , ω and ρ are nonzero real-valued constants such that κ is the frequency of the soliton, ω is the wave number and ρ is the velocity soliton. The functions $\varphi_j(\xi)$ for $j = 1,2$ are real functions which represent the amplitude portions of the solitons and the phase components of the solitons, respectively. Inserting (3.1) and (3.2) into equations (2.2) and (2.3) gives the real parts:

$$(-\rho b_1 + a_1 + h_1)\varphi_1'' + [c_1 - \kappa(r_1 + \mu_1)]\varphi_1^3 + f_1\varphi_2^2\varphi_1^3 + d_1\varphi_2^2\varphi_1 + e_1\varphi_1^5 + g_1\varphi_2^4\varphi_1 + [-\kappa^2 a_1 + \kappa((-\sigma^2 + \omega)b_1 - \lambda_1) + \sigma^2 - \omega]\varphi_1 - \varphi_2 Q_1 = 0 \quad (3.4)$$

And

$$(-\rho b_2 + a_2 + h_2)\varphi_2'' + [c_2 - \kappa(r_2 + \mu_2)]\varphi_2^3 + f_2\varphi_1^2\varphi_2^3 + d_2\varphi_1^2\varphi_2 + e_2\varphi_2^5 + g_2\varphi_1^4\varphi_2 + [-\kappa^2 a_2 + \kappa((-\sigma^2 + \omega)b_2 - \lambda_2) + \sigma^2 - \omega]\varphi_2 - \varphi_1 Q_2 = 0 \quad (3.5)$$

while the imaginary parts are:

$$[3\mu_1 + 2\theta_1 + r_1]\varphi_1'\varphi_1^2 + [(-\rho b_1 + 2a_1)\kappa + (\sigma^2 - \omega)b_1$$

$$+\lambda_1+\rho]\varphi_1^{'}=0, \quad (3.6)$$

And

$$[3\mu_2+2\theta_2+r_2]\varphi_2^{'}\varphi_2^2+[(-\rho b_2+2a_2)\kappa+(\sigma^2-$$
$$-\omega)b_2+\lambda_2+\rho]\varphi_2^{'}=0, \quad (3.7)$$

The linearly independent principle is applied on (3.6) and (3.7) to get the wave number ρ :

$$\rho=\frac{(\sigma^2-\omega)b_j+2a_j\kappa+\lambda_j}{\kappa b_j-1}, \quad (3.8)$$

and

$$3\mu_j+2\theta_j+r_j=0,$$

provided $\kappa b_j\neq 1$ where $j=1,2$.
Now, let us set

$$\varphi_2=Z\varphi_1, \quad (3.9)$$

where Z is a nonzero constant, such that $Z\neq 1$. equations (3.4) and (3.5) can be reduced as:

$$(-\rho b_1+a_1+h_1)\varphi_1^{''}+(Z^2f_1+Z^4g_1+e_1)\varphi_1^5$$
$$+[Z^2d_1-(r_1+\mu_1)\kappa+c_1]\varphi_1^3-[\kappa^2a_1+((\sigma^2-\omega)b_1$$
$$+\lambda_1)\kappa+ZQ_1-\sigma^2+\omega]\varphi_1=0 \quad (3.10)$$

and

$$Z(-\rho b_2+a_2+h_2)\varphi_1^{''}+(Z^3f_2+Z^5e_2+Zg_2)\varphi_1^5$$
$$+\big[\big((-r_2-\mu_2)\kappa+c_2\big)Z^3+Zd_2\big]\varphi_1^3-[Z(\kappa^2a_2+((\sigma^2$$
$$-\omega)b_2+\lambda_2)\kappa+\omega-\sigma^2)+Q_2]\varphi_1=0 \quad (3.11)$$

equations (3.11) and (3.12) are equivalent under the constraint conditions:

$$-\rho b_1+\ a_1+h_1=Z(-\rho b_2+a_2+h_2),$$
$$Z^2f_1+Z^4g_1+e_1=Z(Z^2f_2+Z^4e_2+g_2),$$
$$Z^2d_1-(r_1+\mu_1)\kappa+c_1$$
$$=Z\big[\big((-r_2-\mu_2)\kappa+c_2\big)Z^2+d_2\big],$$
$$\kappa^2a_1+\big((\sigma^2-\omega)b_1+\lambda_1\big)\kappa+ZQ_1-\sigma^2+\omega$$
$$=Z[\kappa^2a_2+(1+(\sigma^2-\omega)b_2+\lambda_2)\kappa+\omega-\sigma^2]+Q_2. \quad (3.12)$$

From (3.12), the soliton velocity is yielded as:

$$\omega=\frac{\begin{bmatrix}Q+\sigma^2-\kappa(\lambda_1+\sigma^2b_1)-ZQ_1+Z[-\sigma^2\\ \kappa(\lambda_2+\sigma^2b_2)+\kappa^2a_2]-\kappa^2a_1\end{bmatrix}}{-\kappa b_1+Z(\kappa b_2-1)+1}, \quad (3.13)$$

provided $-\kappa b_1+Z(\kappa b_2-1)+1\neq 0$.
In order to solve equation (3.10), it can be rewritten as follows:

$$\varphi_1^{''}+l_1\varphi_1+l_2\varphi_1^3+l_3\varphi_1^5=0, \quad (3.14)$$

where

$$l_1=\frac{-\big[\kappa^2a_1+\big((\sigma^2-\omega)b_1+\lambda_1\big)\kappa+ZQ_1-\sigma^2+\omega\big]}{-\rho b_1+a_1+h_1},$$

$$l_2=\frac{Z^2d_1-(r_1+\mu_1)\kappa+c_1}{-\rho b_1+a_1+h_1},$$

$$l_3=\frac{Z^2f_1+Z^4g_1+e_1}{-\rho b_1+a_1+h_1}, \quad (3.15)$$

provided $-\rho b_1+a_1+h_1\neq 0$.
In the following section, we will discover the optical solitons of equations (2.1) and utilizing the well-known enhanced direct algebraic method.

4. An improve direct algebraic approach

Let us now, solve equation (3.14) under the constraint conditions (3.15) as follows: First, balancing $\varphi_1^{''}(\xi)$ and $\varphi_1^5(\xi)$ in equation (3.14) gives $N=\frac{1}{2}$. Therefore, the new wave transformation:

$$\varphi_1=[H(\xi)]^{\frac{1}{2}}, \quad (4.1)$$

where $H(\xi)$ is a new function of ξ, such that $H(\xi)>0$, changes equation (3.14) to the following new nonlinear ODE:

$$2HH^{''}-H^{'2}+4H^2(l_3H^2+l_2H+l_1)=0.) \quad (4.2)$$

The improve direct algebraic approach introduced by Arnous et al. [19] supposes that equation (4.2) has the following formal solution:

$$H(\xi)=\sum_{k=0}^{N}A_k\psi^k(\xi), \quad (4.3)$$

where A_0, A_k $(k=1,2,\ldots,N)$ are constants such that $A_N\neq 0$. While the function $\psi(\xi)$ holds the nonlinear ODE:

$$\psi'^2(\xi)=\sum_{j=0}^{4}\tau_j\psi^j(\xi), \tag{4.4}$$

where τ_j ($j=0,\ 1,\ 2,\ 3,\ 4$) are constants provided $\tau_4\neq 0$. It is well-known [19] that equation (4.4) has many types of exact solutions.

Now, balancing the terms HH'' and H^4 in equation (4.4), yields the balance number $N=1$. Consequently, the formula solution (4.2) takes the following form:

$$H(\xi)=A_0+A_1\psi(\xi), \tag{4.5}$$

where $A_1\neq 0$.

By evaluating (4.5) along with (4.4) into equation (4.2) and utilizing Maple, there are many families of results that will be discussed as follows:

Family-1. When $\tau_0=\tau_1=\tau_3=0$, we have the following results:

Result 1.

$$A_0=0,\ A_1=\sqrt{-\frac{3\tau_4}{4l_3}},\ l_1=-\frac{\tau_2}{4},\ l_2=0, \tag{4.6}$$

provided $l_3\tau_4<0$.

By substituting (4.6) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), two types of soliton solutions can be derived as follows:

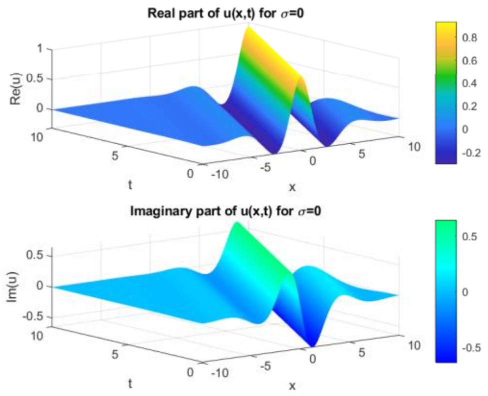
I. If $\tau_2>0$ and $\tau_4<0$, then we have the bright soliton solutions

$$u(x,t)=\left\{\sqrt{\frac{3\tau_2}{4l_3}}sech\left(\sqrt{\tau_2}\xi\right)\right\}^{\frac{1}{2}}$$

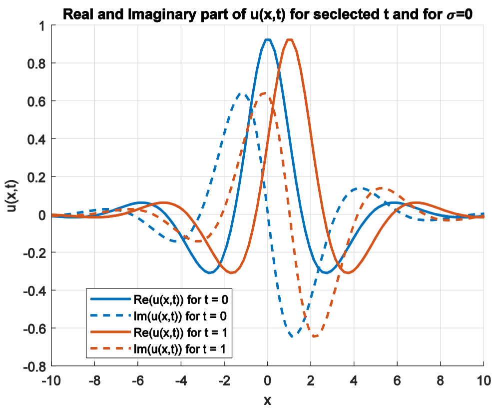
$$\times e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.7}$$

$$v(x,t)=Zu(x,t), \tag{4.8}$$

provided $l_3>0$.



(a)



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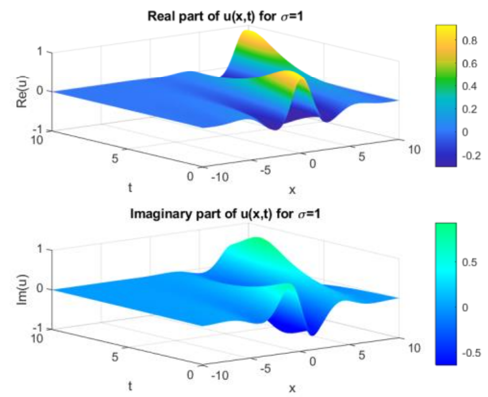

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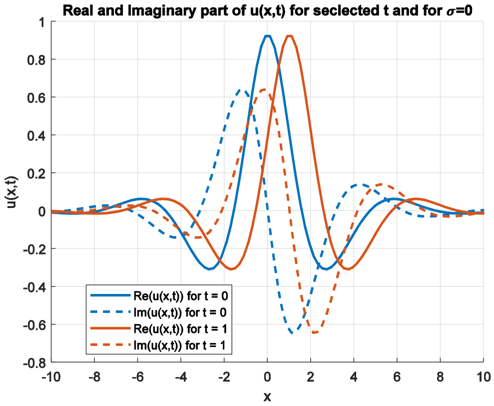


(d)

Figure 1. shows the numerical solution (4.7) in 3D and 2D plots with : $\tau_2 = 1$, $l_3 = \kappa = \omega = 1$, $W(t) = \sqrt{t}$ and the white noise coefficient σ are shown in graphs (a)-(d).

II. If $\tau_2 > 0$ and $\tau_4 > 0$, then we have the singular soliton solutions

$$u(x,t) = \left\{ \sqrt{-\frac{3\tau_2}{4l_3}} csch\left(\sqrt{\tau_2}\xi\right) \right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x + (\omega - \sigma^2)t + \sigma W(t)]}, \quad (4.9)$$

$$v(x,t) = Zu(x,t), \quad (4.10)$$

provided $l_3 < 0$.

Result 2.

$$A_0 = \sqrt{\frac{3\tau_2}{4l_3}},\ A_1 = \sqrt{-\frac{3\tau_4}{4l_3}},\ l_1 = \frac{5\tau_2}{4},$$

$$l_2 = -\sqrt{\frac{16\tau_2 l_3}{3}}. \quad (4.11)$$

This result leads to the bright soliton solutions

$$u(x,t) = \left\{ \sqrt{\frac{3\tau_2}{4l_3}} \left(1 + sech\left(\sqrt{\tau_2}\xi\right)\right) \right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x + (\omega - \sigma^2)t + \sigma W(t)]}, \quad (4.12)$$

$$v(x,t) = Zu(x,t), \quad (4.13)$$

provided $\tau_2 > 0$, $\tau_4 < 0$ and $l_3 > 0$.

Family-2. When $\tau_0 = \frac{\tau_2^2}{4l_4}$, $\tau_1 = \tau_3 = 0$, $\tau_2 < 0$ and $\tau_4 > 0$, we have the following result:

$$A_0 = \sqrt{\frac{3\tau_2}{8l_3}}, A_1 = \sqrt{-\frac{3\tau_4}{4l_3}}, l_1 = \frac{\tau_2}{2},$$

$$l_2 = -\sqrt{\frac{8\tau_2 l_3}{3}}, \quad (4.14)$$

provided $l_3 < 0$.

By substituting (4.14) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), we obtain the dark soliton solutions:

$$u(x,t) = \left\{ \sqrt{\frac{3\tau_2}{8l_3}} \left(1 + tanh\left(\sqrt{-\frac{\tau_2}{2}}\xi\right)\right) \right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x + (\omega - \sigma^2)t + \sigma W(t)]}, \quad (4.15)$$

$$v(x,t) = Zu(x,t), \quad (4.16)$$

and the singular soliton solutions

$$u(x,t) = \left\{ \sqrt{\frac{3\tau_2}{8l_3}} \left(1 + coth\left(\sqrt{-\frac{\tau_2}{2}}\xi\right)\right) \right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x + (\omega - \sigma^2)t + \sigma W(t)]}, \quad (4.17)$$

$$v(x,t) = Zu(x,t), \quad (4.18)$$

Family-3. When $\tau_1 = \tau_3 = 0$, $\tau_0 = \frac{m^2(1-m^2)\tau_2^2}{(2m^2-1)^2\tau_4}$, $\tau_2 > 0$, $\tau_4 < 0$ and $0 < m < 1$, we have the following result:

$$m = 1,\ A_0 = 0,\ A_1 = \sqrt{-\frac{3\tau_4}{4l_3}},\ l_1 = -\frac{\tau_2}{4},$$

$$l_2 = 0, \quad (4.19)$$

Provided: $l_3 < 0$.

By substituting (4.19) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), we have the same bright-soliton solutions (4.9)

Family-4. When $\tau_1 = \tau_3 = 0$, $\tau_0 = \frac{m_2^2\tau_2^2}{(m^2+1)^2\tau_4}$, $\tau_2 < 0$, $\tau_4 > 0$ and $0 < m < 1$, we have the following result:

$$A_0 = -\frac{2m^2\tau_2}{l_2(m^2+1)},\ A_1 = \sqrt{-\frac{4\tau_2\tau_4 m^2}{l_2^2(m^2+1)}},$$

$$l_1 = \frac{\tau_2(5m^2-1)}{4m^2+4},\ l_3 = \frac{3l_2^2(m^2+1)}{16m^2\tau_2}. \quad (4.20)$$

By substituting (4.20) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), we have the Jacobi elliptic function solutions:

$$u(x,t) = \left\{-\frac{2m^2\tau_2}{l_2(m^2+1)}\left(1 + JacobiSN\left(\sqrt{-\frac{\tau_2}{m^2+1}}, m\right)\right)\right\}^{\frac{1}{2}}$$

$$\times\ e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \qquad (4.21)$$

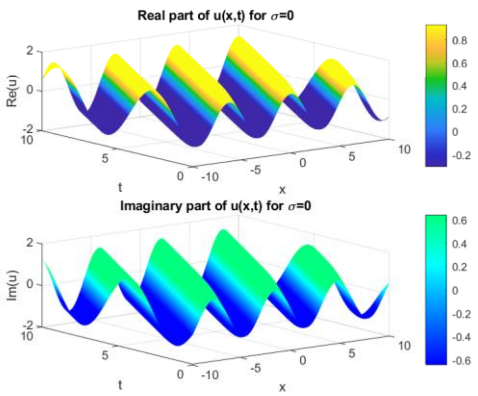
$$v(x,t) = Zu(x,t), \qquad (4.22)$$

Remark In particular, if $m = 1$, the Jacobi elliptic function solutions (4.21) and (4.22) can be converted to the dark solitons

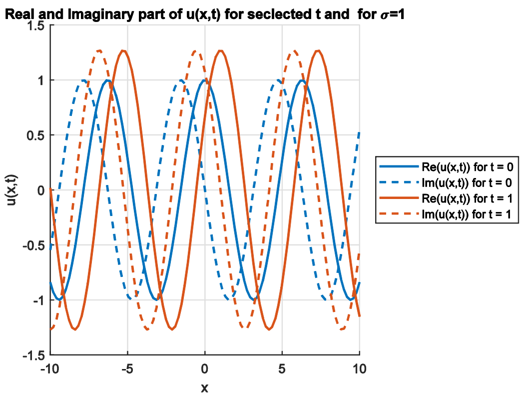
$$u(x,t) = \left\{-\frac{\tau_2}{l_2}\left(1 + tanh\left(\sqrt{-\frac{\tau_2}{2}}\right)\right)\right\}^{\frac{1}{2}}$$

$$\times\ e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \qquad (4.23)$$

$$v(x,t) = Zu(x,t), \qquad (4.24)$$



(e)



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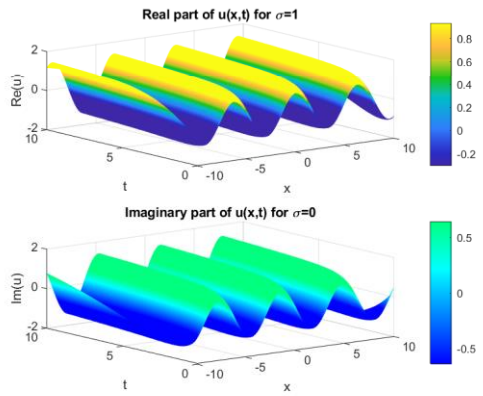
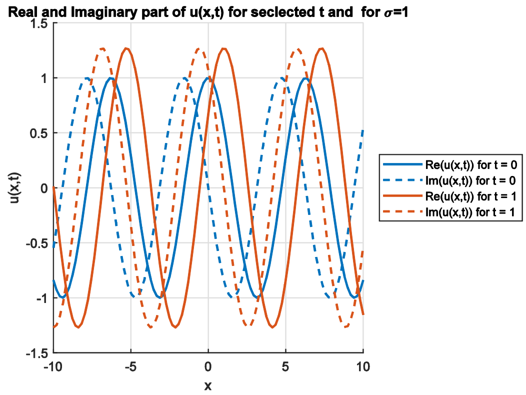


(g)



(h)

Figure 2. shows the numerical solution (4.23) in 3D and 2D plots with : $\tau_2 = -1$, $l_3 = \kappa = \omega = 1$, $W(t) = \sqrt{t}$ and the white noise coefficient σ are shown in graphs (a)-(d).

Family-5. When $\tau_1 = \tau_3 = 0$, $\tau_0 > 0$ and $\tau_4 > 0$ we have the following result:

$$A_0 = \frac{2\sqrt{\tau_0\tau_4}}{l_2},\ A_1 = \frac{2\tau_0^{\frac{1}{4}}\tau_4^{\frac{3}{4}}}{l_2},\ l_1 = -\sqrt{\tau_0\tau_4},$$

$$l_3 = -\frac{3l_2^2}{16\sqrt{\tau_0\tau_4}}, \tau_2 = -2\sqrt{\tau_0\tau_4}, \qquad (4.25)$$

By substituting (4.25) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), we have the Weierstrass-elliptic function solutions:

$$u(x,t) = \left\{\frac{2(\tau_0\tau_4)^{\frac{1}{4}}}{l_2}\left((\tau_0\tau_4)^{\frac{1}{4}}\right.\right.$$

$$+\frac{3\wp'\left(\xi;\frac{\tau_2^2}{12}+\tau_0\tau_4,\frac{\tau_2(36\tau_0\tau_4-\tau_2^2)}{216}\right)}{6\wp\left(\xi;\frac{\tau_2^2}{12}+\tau_0\tau_4,\frac{\tau_2(36\tau_0\tau_4-\tau_2^2)}{216}\right)+\tau_2}\Bigg)\Bigg\}^{\frac{1}{2}}$$

$$\times\ e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.26}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.27}$$

and

$$u(x,t)=\Bigg\{\frac{2\sqrt{\tau_0\tau_4}}{l_2}\Bigg(1$$

$$+\frac{6(\tau_0\tau_4)^{\frac{1}{4}}\wp\left(\xi;\frac{\tau_2^2}{12}+\tau_0\tau_4,\frac{\tau_2(36\tau_0\tau_4-\tau_2^2)}{216}\right)+\tau_2}{3\wp'\left(\xi;\frac{\tau_2^2}{12}+\tau_0\tau_4,\frac{\tau_2(36\tau_0\tau_4-\tau_2^2)}{216}\right)}\Bigg)\Bigg\}^{\frac{1}{2}}$$

$$\times\ e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.28}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.29}$$

Family-6. When $\tau_0=\tau_1=0$ and $\tau_2>0$, we have the following result:

$$A_0=0,\ A_1=\sqrt{-\frac{3\tau_4}{4l_3}},\ l_1=-\frac{\tau_2}{4},$$

$$l_2=\tau_3\sqrt{-\frac{l_3}{3\tau_4}}, \tag{4.30}$$

provided $l_3\tau_4<0$.

By substituting (4.30) with the well-known solutions of equation (4.4) mentioned in [19] into (4.5) and using (4.1) as well as (3.1), (3.2), many types of straddled soliton solutions can be derived as follows:

$$u(x,t)=\left\{-\sqrt{-\frac{3\tau_4}{l_3}}\left(\frac{\tau_2\,sech^2\left(\frac{\sqrt{\tau_2}\xi}{2}\right)}{4\sqrt{\tau_2\tau_4}\,tanh\left(\frac{\sqrt{\tau_2}\xi}{2}\right)+2\tau_3}\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.31}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.32}$$

$$u(x,t)=\left\{\sqrt{-\frac{3\tau_4}{l_3}}\left(\frac{\tau_2\,csch^2\left(\frac{\sqrt{\tau_2}\xi}{2}\right)}{4\sqrt{\tau_2\tau_4}\,coth\left(\frac{\sqrt{\tau_2}\xi}{2}\right)+2\tau_3}\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.33}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.34}$$

$$u(x,t)=\left\{\sqrt{-\frac{3\tau_4}{l_3}}\left(-\frac{\tau_2\tau_3\,sech^2\left(\frac{\sqrt{\tau_2}\xi}{2}\right)}{2\tau_3^2-2\tau_2\tau_4\left(1-tanh\left(\frac{\sqrt{\tau_2}\xi}{2}\right)\right)^2}\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.35}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.36}$$

and

$$u(x,t)=\left\{\sqrt{-\frac{3\tau_4}{l_3}}\left(\frac{\tau_2\tau_3\,csch^2\left(\frac{\sqrt{\tau_2}\xi}{2}\right)}{2\tau_3^2-2\tau_2\tau_4\left(1-coth\left(\frac{\sqrt{\tau_2}\xi}{2}\right)\right)^2}\right)\right\}^{\frac{1}{2}}$$

$$\times e^{i[-\kappa x+(\omega-\sigma^2)t+\sigma W(t)]}, \tag{4.37}$$

$$v(x,t)=\mathrm{Z}u(x,t), \tag{4.38}$$

5. Conclusions

The well-known improve direct algebraic approach have been employed to find the optical-solitons of the coupled system of perturbed resonant NLSE in magneto-optic waveguides with STD, IMD, parabolic law nonlinearity, and multiplicative noise in the Itô sense. Dark solitons, bright solitons, singular solitons, straddled solitons, Jacobi-elliptic functions solutions and Weierstrass-elliptic functions solutions are reported for the first time. Figures 1 and 2 have presented the numerical simulations of solutions (4.7) and (4.23) with no noises as well as at $\sigma=0$ and $\sigma=1$. In these figures, when the noise vanishes, we note that the surface is less planer, while when the noise increases, we note that surface becomes more planer after small transit behaviors. This means the multiplicative noise effects on the solutions and it makes the solutions stable. Finally, this study has concluded that the noise effect (noise strength) on the soliton solutions has a significant effect.

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