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Proposed Method for Estimating the Number of Sources Arranged in Linear Array via Correlation Analysis

Mohamed.S.Alshulle[a], Lubna.A.Attelisi[b]
[a]Military Industries Organization, Libya
[b]Communication and Microwaves Department, College of Electronic Technology, Bani-Walid Libya
*Crossponding author: dr.engineeralshole@gmail.com

Abstract: For the wireless communication system based on an array antenna with a large number of elements, it is important to accurately estimate the number of signals included in a received signal, because the information for the number of signals is required for determining the minimum number of active elements in the received antenna. [1] Also estimating the number of sources received by an antenna array have been well known and investigated since the starting of array signal processing. Accurate estimation of such parameter is critical in many applications that involve prior knowledge of the number of received signals and array signal processing, also they are important in phased array radar, brain imaging, speech signal separation, and the direction of arrival (DOA) estimation. [2,3] In this paper, we offer an efficient algorithm for estimating the number of signals in a given range: the beamspace based Akaike Information Criterion (AIC) and Minimum Description Length (MDL). Through computer simulation, the suggested algorithm's estimation performance is assessed and examined.

Keywords: Akaikes information criterion, Beamspace, Minimum description length, Estimation.

Introduction

From the start of signal processing, estimating the number of sources has been investigated, since it is very important to many applications to accurately estimate the number of signals included in a received signal. These applications presume that this parameter was known beforehand and that it would be necessary to depend on it for additional processing. [1] Two extensively used and popular methods for calculating the number of signals in wideband communications based on information theory techniques, the Akaike Information Criterion (AIC) and Minimum Description Length (MDL) are representative and superior techniques for determining the number of signals. Despite being more sophisticated than others, they have been utilized frequently. Based on these two algorithms, the number of signal is determined as the value for which the AIC or MDL criterion is minimized [4]. On the other hand, even though these two methods are effective in determining the number of sources they require the estimation of covariance matrix and its eigenvalue decomposition which generally leads to high complexity.

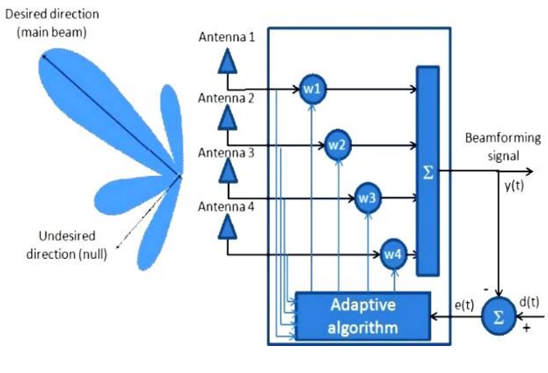

Fig. 1: Adaptive Antenna Array system

Signal Model

The beamspace output signal model and the received signal model, which includes a variety of signals and noise, are presented in this section.

By considering a scenario where p wideband coherent signals impinging on a uniform linear array composed of M sensors. The wideband signals bandwidth should not be identical where the frequency bandwidth is $[f_L, f_H]$ and the signals come from different directions $\theta_p, p = 1, \ldots p$ leading to an output of the array corrupted by additive noise at the m^{th} sensor as follows:

$$X_m(t) = \sum_{p=1}^{p} S_p\left[t - \tau_m(\theta_p)\right] + n_m(t), 1 \le m \le M \quad (1)$$

Where $S_p(t)$ is the p^{th} wideband signal, $n_m(t)$ is the noise corrupted with the signal at the m^{th} sensor and the $\tau_m(\theta_p)$ is the propagation delay correlated with the m^{th} sensor and the p^{th} source which can be defined as: [5]

$\tau_m(\theta_p) = \frac{(\boldsymbol{m-1\_dsin(\theta_p)}}{\boldsymbol{c}}$ (2) where c is the velocity of the propagation.

Consider all sources can be separated into J non overlapping narrowband blocks where the observation time is assumed to be long enough for the Fourier transform of the sensor output to have good resolution. And by using discrete time Fourier transform (DTFT), the array output at a specific frequency can be expressed as:

$$X(f_i) = A(f_i, \theta)S(f_i) + N(f_i), \quad j = 1, \ldots \ldots J \quad (3)$$

Where $f_L \le f_L \le f_H$, and $A(f_i, \theta)$ the $M \times P$ array response matrix.

$$A(f_i, \theta) = [a(f_i, \theta_1), a(f_i, \theta_2), \ldots \ldots a(f_i, \theta_p)] \quad (4)$$

For which the columns are the $M \times 1$ steering vectors therefore

$$a(f_i, \theta_p) = \left[1, e^{\frac{i2\pi f_i d \sin(\theta_p)}{c}}, \ldots \ldots e^{\frac{i2\pi f_i (M-1) d \sin(\theta_p)}{c}}\right]^T \quad (5)$$

is the steering vector at frequency f_i.

And for suitability, we can use $A_J(\theta) and\ a_j(\theta_p)$ instead of $A(f_i, \theta) and\ a(f_i, \theta_p)$ in the following part.

By considering the noise in the previous equations is gaussian white noise with variance σ_n^2 both temporally and spatially, the array covariance matrix at frequency f_i is:

$$R(f_i) = E[X(f_i)X^H(f_i)] = A_j(\theta)R_s(f_i)A_j^H(\theta) + \sigma_n^2 I \quad (6)$$

Where the correlation matrix of signals at frequency f_i can be represented as:

$$R_s(f_i) = E[S(f_i S^H(f_i) \quad (7)$$

And according to the previous equations, the signal subspace matrix$U_s(f_i)$ and the noise subspace matrix $U_n(f_i)$ can be determined from the eigenvalue decomposition of the array covariance matrix:

$$U_s(f_i) = [u_1(f_i), u_2(f_i), \ldots \ldots u_p(f_i)] \quad (8)$$

$$U_n(f_i) = [u_p{+}_1(f_i), u_p{+}_2(f_i), \ldots \ldots u_M(f_i)] \quad (9)$$

For which $U_i(f_i), i = 1, \ldots \ldots M$ are the orthogonal eigenvectors of $R(f_i)$ [5].

AIC and MDL Based on Theoretic Criteria

In this section we present two important algorithms (AIC & MDL) which are based on theoretic criteria that are used to efficiently estimate the number of non-coherent signals. The eigenvalues of the sample auto covariance are used in these order determination information theoretic models to calculate the number of approximately equal smallest eigenvalues. Certain eigenvalues would be located in the signal subspace, while others in the noise subspace. The goal of both methods is to minimize a log likelihood criterion over the total number of detectable signals [2].

According to these two algorithms, simulation results that show their performance will be illustrated with more details.

1- Signal Model

The information theoretic criteria for model selection, introduced by Akaike Schwartz states to select the model which gives the minimum AIC that can be defined in the following equation:

$$AIC = -2\log f(X|\hat{\theta}) + 2k \quad (10)$$

For which the maximum likelihood estimation of the parameter vector θ is represented as $\hat{\theta}$ in the past equation, and k is the number of free adjusted parameters in θ [4].

The ways in which Schwartz and Rissanen tackled the issue were very unlike. The foundation of Rissanen's strategy is information theory. Rissanen suggested choosing the model that produces the minimum code length because all of the models can be used to encode the observed data. Overall, both Schwartz's and Rissanen's approaches lead to the same criterion, given by:

$$MDL = -log f(X|\hat{\theta}) + \frac{1}{2}\, k \log N \quad (11)$$

And to determine the number of signals, we apply the information criteria.

$$f(x(t_1),\dots,x(t_N)|\theta^k)\ \prod_{i=1}^{N}\frac{1}{\pi^p \det R^k}\exp - x(t_i)^{\dagger}[R^k]^{-1}x(t_i)$$
(12)

The next step will depend on removing the terms that do not depend on the parameter vector θ^k and taking the logarithm, we than can find the log-likelihood function $L(\theta^k)$ as:

$$L(\theta^k) = -n\log\det R^k - tr[R^k]^{-1}\hat{R} \quad (13)$$

What maximizes (13) is the value of θ^k that represents the maximum-likelihood estimate, where these estimates can be defined as:

$$\hat{V}_i = c_i; i = 1,\dots,k \quad (14)$$

$$\hat{\lambda}_i = l_i; i = 1,\dots,k \quad (15)$$

$$\hat{\sigma}_n^2 = \frac{1}{M-k}\sum_{i=k+1}^{M} l_i \quad (16)$$

We now substitute the maximum likelihood estimates (8) in (13), we obtain the following equations with applying some straightforward manipulations.

$$L(\hat{\theta}) = \log(\frac{\prod_{i=k+1}^{M} l_i^{\frac{1}{(M-k)}}}{\frac{1}{(M-k)}\sum_{i=k+1}^{M} l_i}\ (M-k)N \quad (17)$$

After applying the previous equations, we can obtain the form of both AIC and MDL equation [4].

$$AIC(k) = -2N\ \ln\left[\frac{\prod_{i=K+1}^{M} a_i}{[\frac{1}{M-K}\sum_{i=K+1}^{M} a_i]^{M-K}}\right] + 2k(2M-K)$$
(18)

$$MDL(k) = -N\ln\left[\frac{\prod_{i=K+1}^{M} a_i}{[\frac{1}{M-K}\sum_{i=K+1}^{M} a_i]^{M-k}}\right] + \frac{1}{2}K(2M-K)\ln N$$
(19)

The signal-to-noise ratio (SNR) level and the number of samples are two examples of scenarios where the estimate capabilities of the AIC and MDL methods may differ, hence it is best for these two algorithms to be completely employed to each other [1].

Simulation Results

In this section, computer simulation results have been carried out to verify the effectiveness of the proposed AIC and MDL algorithms for which all the following results are simulated using MATLAB. For this simulation, we assumed that a uniform linear array with 6 sensors is used, 11 array elements with a signal to noise ration 10db and 128 as a value for the number of snapshots taken.

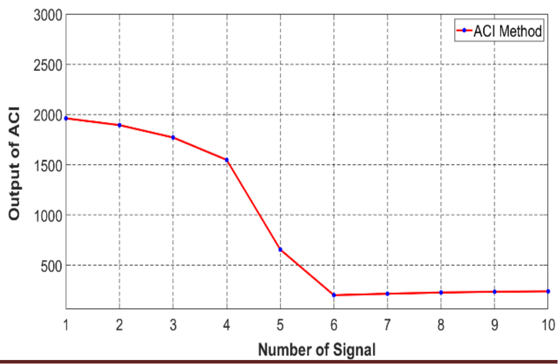

Fig. 2: Estimating the Number of Sources Using AIC

Here AIC method was used and as the simulation results show, the values of the estimated sources are close to each other and difficult to distinguish between the output of ACI at 6 and 7 signals.

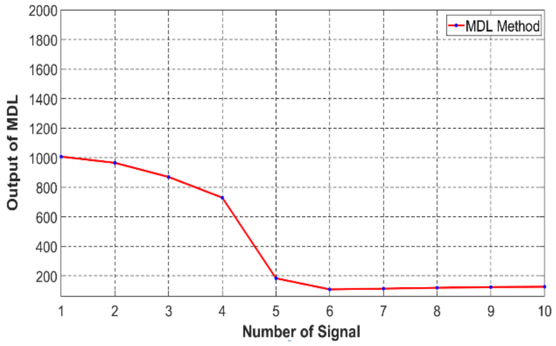

Fig. 3: Estimating the Number of Sources Using MDL

In MDL it is clearer that the results gave better value in terms of estimating the signals, where the difference between the values of the output is easy to differ them from each other.

Table 1: AIC and MDL output.

Number of Sources	AIC	MDL	Index Minimum Value
1	1935	1041	
2	1854	976.7	
3	1710	897	
4	1498	737.9	
5	778.1	180.7	
6	202.3	107.2	6
7	216.5	114	
8	227.7	119.4	
9	236.4	123.6	
10	236.4	126.4	

Correlation Analysis

According to the covariance matrix R_x and after computing the eigendecomposition we can conclude that the eigenvalues of R_x have the following property:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D \geq \lambda_{D+1} = \cdots \lambda_M = \sigma_n^2 \qquad (20)$$

For which D is considered as the larger eigenvalues of R_x associated with signals and $M - D$ are the smaller eigenvalues of R_x associated with the noise, and the signal and the noise are simultaneously corresponding to the eigenvectors of matrix x R that pertain to these eigenvalues. Consequently, these eigenvalues (also known as eigenvectors) can be further subdivided into signal and noise eigenvalues.

Through eigendecomposition of the covariance matrix R_x we compute the eigenvalues of R_x in decreasing order.

$$[U\ S\ V] = svd(R_x) \qquad (11)$$

$$S = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_d & 0 & 0 \\ 0 & 0 & 0 & \lambda_{d+1} & 0 \\ 0 & 0 & 0 & 0 & \lambda_M \end{bmatrix} \qquad (12)$$

For which S is considered to be the diagonal of R_x eigenvalues.

If we look back to MDL equation in (19) we can assume that it's divided into two parts:

$$part\ 1 = -N \ln \left[\frac{\prod_{i=K+1}^{M} a_i}{\left[\frac{1}{M-K}\sum_{i=K+1}^{M} a_i\right]^{M-K}} \right] \qquad (13)$$

$$part\ 2 = \frac{1}{2} K(2M - K) \ln N \qquad (14)$$

If we analyze both equations separately, we can see that part 1 is a composite of a numerator that is similar to the denominator in terms of equation synthesis, therefore it will not affect the equation significantly, however part 2 is precisely the opposite, where applying different values to this equation gives completely different results and far apart from each other. And since signals and noise are both combined together to form the eigenvalues, we can use the second part of the MDL equation in the following analysis in order to distinguish between the values of the signals and noise from the eigenvalues in an automatic process through simulation.

After determining the eigenvalues, we can notice from the values that there is not a big difference between the last value for the signal and the first value for the noise, which makes it so difficult to separate them from each other unless it's done manually. Consequently, if we take the logarithm for each of them, we will be able to bring the values closer together and make it easier to differentiate between signals and noise.

$$E_n = \log(aa^2) \quad (15)$$

To show the results in a stronger way we multiply (15) in $(M-1)$ factor to make the results dependent on the number of elements, which means that by increasing the number of elements it will help to determine the number of signals.

$$E_n = 0.5 \times (M-1) \times \log_{10}(aa^2) \quad (16)$$

By implementing the previous analysis, we can set the threshold value as 1 therefore, all the values above 1 are considered as signals and the values below is noise.

Conclusion

In this paper, we concentrated on assessing the effectiveness of two widely used techniques, AIC and MDL. These methods work well for identifying the number of non-coherent signals, as demonstrated by the simulation results, and their effectiveness will decline for coherent and/or correlated signals, however one way was more efficient than the other, for which MDL was more effective in estimating the number of sources. In general, both methods have the ability in estimation and give close results to each other therefore if both cases gave exactly the same results, it is better to take the results of MDL and if there was a slight difference between them, we take the method that gave us the more enhanced and clear results. After estimating the number of sources, a specific analysis was used in order to separate the eigenvalues into signals and noise and this process was done dependent on M values which will lead to bigger results and makes it easy to distinguish the signals from the eigenvalues.

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