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Enhanced MUSIC Approach for Estimation of Directions of Arrival for Multiple Coherent Wideband Sources

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Abstract: Direction of Arrival (DOA) is a term used in signal processing literature to describe the direction from which a propagating wave typically arrives at a place, which is usually where a set of sensors are located. Many signal processing applications need the estimation of a set of unknown characteristics based on data gathered from a variety of sensors. Electronic surveillance, sonar, and radar are only a few of many applications that require high resolution DOA estimate and among these applications, DOA estimation of narrow band signals is one that has drawn a lot of interest from researchers. The most promising class of approaches among those suggested to solve the DOA problem is signal subspace algorithms. These techniques attempt to divide the space spanned by the measured data into subspaces known as signal and noise by taking advantage of the underlying data model. The approach that has drawn the greatest interest and been examined the most within this class of algorithms The Multiple Signal Classification (MUSIC) technique can be used with arbitrary geometry [1]. This paper focusses on estimating the DOA using MUSIC method but one of the main issues in this method is its inability to estimate highly correlated signals due to loss of non-singularity property of the covariance matrix that is used in the signal model [2], therefore a developed method (Focused Method) will be proposed in this paper and used to overcome this problem. The direction of arrival estimation is simulated on a MATLAB software with a set of input parameters such as array elements, signal to noise ratio, number of snapshots and number of signal sources. An extensive simulation has been conducted and the results show that the MUSIC can achieve an accurate and efficient DOA estimation in coherent cases.

Keywords: Direction of arrival (DOA), estimation, Multiple Signal Classification (MUSIC), Focused Method.

Introduction

In the past decades, many array models such as the uniform linear array (ULA), uniform rectangular array, uniform circular array, and non-uniform linear array (NLA) have been employed towards the achievement of the direction of arrival (DOA) estimation [3]. And by achieving an accurate DOA estimation of a signal sent from a source to a receiving antenna can increase the wireless communication system capacity. Therefore, working to advance DOA estimate techniques is a key to improving the quality of wireless networks [3]. In recent years DOA estimate has developed dramatically and it also has numerous further uses, such as object tracking and localization dedicating the signal to a desired user in wireless networks and sound and speech processing hence many methods have been employed in these applications and to solve such problems related to DOA, among them MUSIC method for which it's also classified to be one of the most popular techniques used in DOA estimation due to its accuracy, high resolution and stability under certain conditions [2].

MUSIC is a DOA estimating technique is based on using eigenspace method to estimate the DOA content of a signal or autocorrelation matrix. It's a type of method that is assumed to be relatively simple and efficient eigenstructure where it has many variations. Using the supplied data, the approach calculates will get the signal subspace which is corresponding to the signal component and the noise subspace which is orthogonal to the noise component, using either singular value decomposition of the data matrix, with its N columns representing the N snapshots of the array signal vectors, or eigenvalue decomposition of the predicted array correlation matrix can be used to do this [1]. After the estimation of the noise subspace and by utilizing the orthogonality of the two subspaces, we could build the spectral function and, by searching for the spectrum peak, determine the direction of the signal incidence, ultimately obtaining the signal high resolution parameter estimation [4].

Even though MUSIC algorithm can estimate independent signals DOA effectively, but it is failure to coherent signals where it gives ineffective results. Consequently, research has focused on determining the coherent signal's direction, and numerous modified MUSIC algorithms have been proposed to enhance the estimation of the coherent signal's performance. These algorithms involve preprocessing the data to reduce or eliminate the correlation between sources [4].

This paper modifies the traditional MUSIC method utilizing the idea of Focused Method for which it makes further improvement on the covariance matrix in order to be able to estimate correlated signals comparing it to the basic idea of MUSIC algorithm. Simulation results of the two algorithms will be presented and compared to each other to show the effectiveness of each method.

Wideband Music Beamformer

The signal and noise subspaces of the observation space have been divided by a number of subspace decomposition techniques. These methods begin with estimating the signal and noise subspaces by the decomposition of the array correlation matrix. Signal subspace is the subspace that the Covariance matrix's eigenvectors that correspond to dominating Eigen-values span. In this paper MUSIC method will be used since it can be characterized as the method for estimating the individual frequencies of multiple time-harmonic signals. The output covariance matrix is used by all traditional beamforming algorithms to estimate the source's direction of arrival. The covariance matrix's ability to display output power at each sensor and its ease of modification to achieve gains at each sensor is another crucial feature.

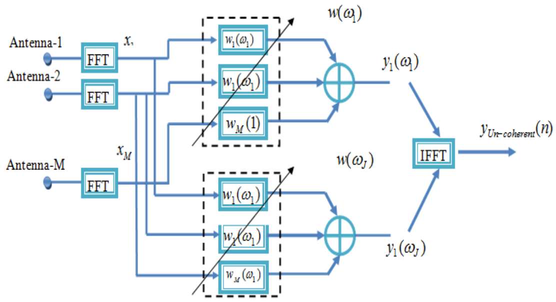

Fig. 1: Block Diagram of the Incoherent Wideband Adaptive Beamformer

Music Algorithm

Multiple signal classification is a subspace method that makes use of the input covariance matrix's eigenstructure. The DOA is estimated with high resolution using a set of input parameters, including the number of array elements, number of snapshots, element spacing, angular separation, signal-to-noise ratio, and MUSIC method [3]. The direction of sources is calculated from steering vectors that orthogonal to the noise subspace. Which finds the peak in spatial power spectrum [5].

1- Signal Model

By assuming that a set of incident signal collected by all antenna at time and are independent of noise:

$$X(t) = As(t) + n(t) \quad (1)$$

And the input correlation matrix of these signals can be expressed as:

$$R_x = E[XX^H] \quad (2)$$

Where H is the conjugate transpose matrix and the noise is assumed to be zero mean for which the additive white Gaussian is uncorrelated to the signal.

By substituting 1 by 2, the covariance matrix results to be:

$$R_x = E[A_s + N)(A_s + N)^H] \quad (3)$$

$$= AE[ss^H]A^H + E[NN^H] \quad (4)$$

$$= AR_SA^H + R_N \quad (5)$$

For which the source signal correlation in the previous equation is defined as $R_s = E[ss^H]$, and the noise correlation matrix is $R_N = \sigma^2 I$.

If we assumed that $(\lambda_1, \lambda_2, \dots \lambda_M)$ are the eigenvalues of the spatial correlation matrix R_x, therefore the performance of eigenvalue related with a particular eigenvector can be obtained as [3]:

$$R_x - \lambda_i I = 0 \quad (6)$$

$$AR_sA^H + \sigma^2 I - \lambda_i I = 0 \quad (7)$$

$$AR_sA^H + (\sigma^2 - \lambda_i)I = 0 \quad (8)$$

According to these equations, eigenvectors v_i of AR_sA^H are determined using:

$$v_i = \sigma^2 - \lambda_i \quad (9)$$

And if we analyze the properties of the previous spatial correlation matrix R_x, it is evident that when there are more array sensors than signal sources, $(M > K)$, when R_s is positive definite, which means that the signals are not fully correlated and the matrix $R_x - \sigma^2 I$ will have rank K and a null space of dimension $M - K$ therefore, the matrix $R_x$will have K eigenvalues greater than σ^2. Also, these eigenvalues may be sorted from the largest to the smallest $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M \geq 0$ where the eigenvectors $\{e_1, e_2, \dots e_k\}$ are corresponding to the largest eigenvalues that identify the signal subspace and the eigenvectors $\{e_{k+1}, e_{k+2}, \dots e_M\}$ are corresponding to the smallest eigenvalues that identify the noise subspace.

By performing orthogonality between different candidate steering vectors and the noise subspace MUSIC special spectrum for wideband signals can be obtained as:

$$p_{music}(\omega, \theta) = \frac{1}{a^H(\omega,\theta)E_N E_N^H\, a(\omega,\theta)} \quad (10)$$

For ideal cases $P_{music}(\theta)$ will peak to infinity when a true $\theta_i, i = 1,2, \dots k$ angle is tested, which means a direction of a specific signal is defined.

The MUSIC technique is only effective when the signals are uncorrelated and that when the rank of the matrix $R_x - \sigma^2 I$ is equal to K.

2- Simulation Results

To evaluate the MUSIC algorithm's performance, a computer application is built using MATLAB. The MUSIC algorithm is applied to signals that has $(\theta = -40,0,40)$ as values with SNR equal to 20. Subsequently in order to detect the DOA estimates, the known/estimated DOAs are applied to the MUSIC Beam former. The results presented in the following figure shows very clearly that MUSIC algorithm is able to estimate the DOA when the incident signals are assumed to be uncorrelated.

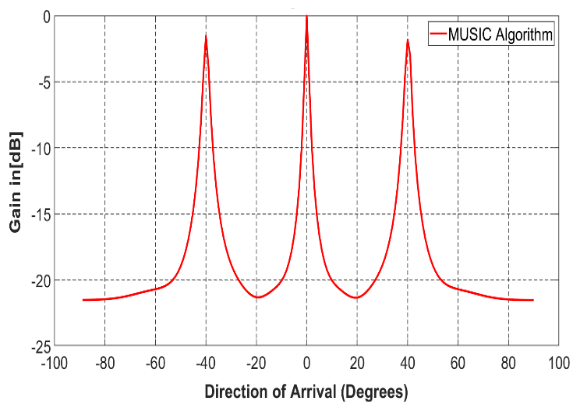

Fig. 2: DOA Estimation using MUSIC Algorithm for Uncorrelated Signals

However, in the other case when the signals are assumed to be correlated for same values of signals, MUSIC algorithm did not effectively estimate the direction of signals as shown in the following figure, for which it shows that the response is not sharp at the peaks while it was sharp in case of uncorrelated input signal condition. And as stated before the source covariance matrix does not satisfy the full rank requirements needed by the MUSIC for Eigen decomposition, which leads to this issue.

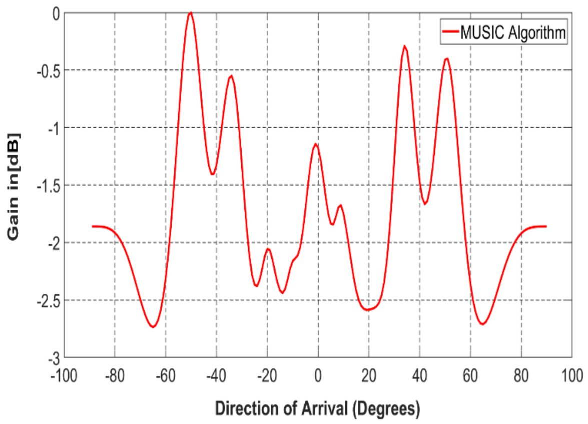

Fig. 3: DOA Estimation using MUSIC Algorithm for Correlated Signals

Focused Method

As the previous results show, we can confirm that the MUSIC algorithm is limited to the estimate of the spatial spectrum for signals that are not coherent. When signal sources are coherent, the coherent signals combine to form a single signal. The array then receives less independent signal sources, which decreases the rank of the array covariance matrix and reduces the number of bigger eigenvalues which makes them less than the number of incoming signals. And to obtain the correct DOA estimation, the spatial spectral curve needs to be present at the peak and as fig (3) shows MUSIC algorithm wasn't able to reach it, therefore to estimate the coherent signal DOA accurately, the correlation between signals needs to be removed.

1- Signal Model

By applying the eigendecomposition of the covariance matrix R_x that is mentioned previously in (3) we compute the eigenvalues of R_x in decreasing order.

$$[U\ S\ V] = svd(R_x) \tag{11}$$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D \geq \lambda_{D+1} = \cdots \lambda_M = \sigma_n^2 \tag{12}$$

$$S = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_d & 0 & 0 \\ 0 & 0 & 0 & \lambda_{d+1} & 0 \\ 0 & 0 & 0 & 0 & \lambda_M \end{bmatrix} \tag{13}$$

Where S is diagonal of R_x eigenvalues.

By assuming λ_i is the i^{th}, $i = D+1, D+2, \dots, M$ eigenvalue of matrix R_x, and eigenvector is v_i we obtain:

$$R_x v_i = \lambda_i v_i \tag{14}$$

And since were working on modifying MUSIC algorithm by using Focusing method we can consider R as:

$$R_{foc} = T_{auto} \times R_x \times T_{auto} \tag{15}$$

For which $T_{auto} = S \times S$ (16)

If we consider R_s is R_{foc} in this case, we can combine both (7) and (14) to get:

$$\sigma^2 v_i = \left(A\, R_{foc}\, A^H + \sigma^2 I\right) v_i \tag{17}$$

When (17) right side is expanded and compared to its left, (17) can be rewritten as follows:

$$A R_{foc} A^H v_i = 0 \tag{18}$$

And as shown in the prior equation, since $A^H A$ is the nonsingular matrix, both R_{foc}^{-1} and the matrix $(A^H A)^{-1}$ exist at the same time, which in this case its very clear that the covariance matrix turned out to be a full rank matrix even though the received signals were correlated in the first place.

If we multiply both sides by $R_{foc}^{-1}(A^H A)^{-1}A^H$ we can derive:

$$R_{foc}^{-1}(A^H A)^{-1}A^H A R_{foc} A^H v_i = 0 \quad (19)$$

Consequently, the focusing method equation can be defined as:

$$P_{foc.muisc}(\omega,\theta) = \frac{1}{a^H(\omega,\theta)\,E_n E_n^H\,a(\omega,\theta)} = \frac{1}{\|E_n^H\,a(\omega,\theta)\|^2}$$

(20)

2- Simulation Results

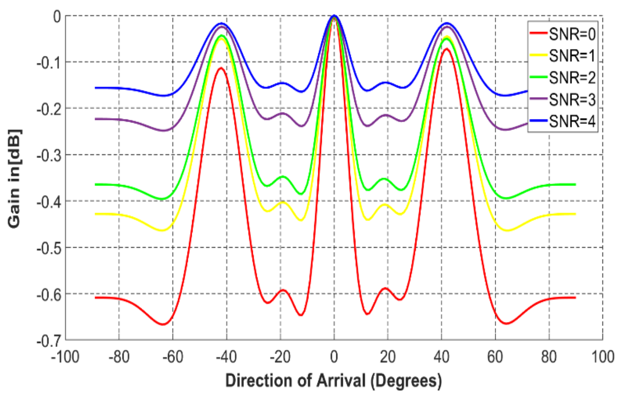

Fig. 4: DOA Estimation using Modified MUSIC Algorithm

These findings demonstrate the focusing method's capacity to estimate the DOA. The SNR is ranged from 0 to 5, and it is evident that a SNR of 0 yields the best results.

Conclusion

In this paper MUSIC Algorithm has been proposed to estimate the DOA of a specific number of signals that are presumed to be uncorrelated, and as the previous results showed, it was able to estimate three different directions of the input signals from the peaks of the spatial spectrum, which are almost the true DOA. However, for the case of correlated signals it was clear that it was efficient enough to estimate the directions of the signals, for which we can summarize that MUSIC method can only be applied for uncorrelated input signals because of the corresponds presence of coherent (correlated) signals is a singular covariance matrix R_x.

Therefore, to overcome this problem a modified MUSIC method is developed. We applied the same signals used in the MUSIC algorithm to be able to compare the results in both methods effectively and it was very clear from the results that the focused method detected the signals direction for which the received signals direction had the sharpest peak comparing it to others.

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