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استخدام القيم الذاتية والمتجهات الذاتية في البيئة

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Utilising Eigenvalues and Eigenvectors in the Environment

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الملخص:

قد يتبع موضوعات التحديد القطري على الفور تنفيذ هذا النموذج، عادة في منتصف الفصل الدراسي في الجامعات يتم تدريس القيم الذاتية والمتجهات الذاتية، وسيرى الطلاب تطبيقات لموضوعات متقدمة معينة مقارنةً بالوحدات الأخرى، وهذا يوضح الاستخدام السريع للقيم الذاتية والمتجهات الذاتية في البحوث البيئية أيضًا.

في حين أن هناك عدة طرق لمحاكاة النمو السكاني، سيتم تقديم مصفوفة من نوع ليزلي في هذا القسم. يتم تصنيف هذا الموضوع بشكل أكثر ملاءمة ضمن "البيئة السكانية.

الكلمات الدالة: القيم الذاتية، المتجهات الذاتية، البيئية، مصفوفة نوع ليزلي، العلوم والتكنولوجيا والهندسة والرياضيات.

Abstract

The subjects of diagonalization may be followed immediately by the implementation of this modulo. Usually in the middle of the semester in university, eigenvalues and eigenvectors are taught. Students will see applications of certain advanced themes in comparison to the other modulo. This illustrates an expeditious use of eigenvalues and eigenvectors in environmental research as well. While there are several methods for simulating population growth, the matrix of the Leslie type will be introduced in this section. This topic is more properly classified under "population ecology".

Keywords: Eigenvalues, Eigenvectors, environmental, Leslie type's matrix, STEM, Mathematics.

1. INTRODUCTION

It is a difficult but crucial issue to connect theory with implementation. All students should be aware of this, but those specialising in STEM education should pay special attention. So that they may succeed in their academic and professional endeavours, we must inspire our engineering students. As our decades of experience instructing mathematics and other STEM-related subjects have demonstrated, inspiring students is inherently a challenging endeavor[1].

This assignment becomes much harder when it comes to STEM education. This is perhaps partly due to the fact that students in STEM-related fields must focus harder and put in more effort to grasp challenging material. Furthermore, the majority of the student groups that we deal with are full-time employees who also have family obligations[2]. A significant portion of these students are members of underrepresented groups and face a variety of personal and professional challenges related to social, economic, and political issues. This is particularly true for evening class students, who usually struggle to focus in class after a long day at work[3

The aforementioned statement is in good agreement with study results.
The following are noted by McKeachie:

Students remember 70% of the information presented in the first ten minutes of a normal fifty-minute lecture lesson, but just 20% in the last ten. We must organise "the material" in a variety of ways to accommodate different student learning styles if we are to effectively convey our message[4]. Complementary remarks are available for Banks, Susan & Linda, Bailey & Alfonso, Ormrod, and Engle & Tinto. Thus, developing modules offers a means of appealing to students' interests while bridging dry theory with exciting practice[5]. Particularly for this module, the System of Linear Equations is being used. This system may be used for a variety of purposes, such as the following:

1. Various real-life C applications represented by natural mathematical models.
2. A rough representation of a non-linear model.
3. Step three involves resolving additional mathematical issues, such as partial differential equations and ordinary equations.

It is difficult to model a population. Conversely, when they study exponential growth and decay in pre-calculus, children are exposed to very specific kinds of modelling at an early age. The size of the population and its pace of growth or decline are, of course, dependent on a variety of real-world conditions[6]. Different "age-groups" and the associated death and reproduction rates are specifically taken into consideration in the Leslie matrix model. Sometimes the age categories are swapped out with various life cycle phases; for instance, we may think of "larva," "toad," etc. in the case of frogs[7-8]. To further understand this procedure, let's begin with an example from a species of bird. Let's start with a pre-calculus warm-up first, however.

2. Educational Results

1. Acquire an understanding of population ecology and dynamics.
2. Use diagonalization to examine population dynamics' long-term behaviour.
3. Recognise the Leslie matrix for the population model.
4. Population model analysis using Eigenvalues and Eigenvectors.

5. Recognise the meanings behind the various components of a Leslie matrix.

6. The computation is performed using Mathematica.

3. Framework for Theory

Before this modulo is used in the classroom, the following subjects will be taught to the students:

1. Eigenvectors and Eigenvalues
2. Split a matrix in half
3. By diagonalizing a matrix, one can ascertain its n^{th} power.

The upcoming module will cover the following subjects[9]. While prior teaching isn't necessary for implementation, it would be advantageous for anyone intending to present these concepts to students[10]. The Eigenvalues and Eigenvectors of a square matrix may be found using "mathematica."

4. Preliminary Polynomial and Exponential Growth as an Example

The following chart shows the total population of a city (referred to as City A in this example) during a 25-year period beginning in 1985. Because population counting is a costly operation, the government only conducted population counts every five years. The number of people is expressed in thousands.

Table 1

1985	1990	1995	2000	2005	2010	2015
13	15	21	32	49	75	110

Over a five-year period, the population measured in thousands. Let's say that t is measured in five years, and that t=0 is the beginning point. Hence, five years after 1985 are indicated by t = 1, ten years after 1985 by t = 2, and so on. In this manner, the measurement and table will line up precisely.

Question no. 1: Utilizing a piece of graph paper, plot the data provided in the preceding table. Designate the y-axis to symbolize the entire population and the x-axis to symbolize time. Therefore, the values for time will be denoted as t = 0, 1, 2, 3, 4, 5, and 6, encompassing seven data points.

Question no. 2: Determine which cubic polynomial suits the provided data best. In other words, the task is to determine the coefficients w, x, y, and z that closely align with the provided data points, beginning with the function $f(t) = wt^3 + xt^2 + yt + z$. Since there are four unknowns, we need at least four equations. Formulate the four equations by substituting the data points for t = 0, 1, 2, and 3 from the table. Proceed to solve this system using Mathematica or any other computational tool. Verify the function's values at t = 4, 5, and 6 to evaluate the accuracy of the estimate.

Question no. 3: Proceed with the same procedure to ascertain the unknown values (w, x, y, and z); however, this time apply the final 4 values (t = 3, 4, 5, and 6). Consequently, a distinct set of linear equations will be obtained. Determine if the resulting function precisely matches the previous one, or if there are slight variations in the values of w, x, y, and z. Verify the function's values for t = 0, 1, and 2.

Question no. 4: Contemplate the application of an exponential model to the given data. Therefore, calculate a function g(t) = ce^{nt}, where c designates the initial population and k signifies the growth rate; in this case, c is obviously twelve thousand. Finding the value of "n" requires only one additional point apart from the initial one. Six distinct models for the exponential function can be derived using the points for t = 1, 2, 3, 5, and 6. Compare the results of each model after including additional values (rounded to the nearest thousand) in the table. To illustrate, when t equals 2, the corresponding value of n= 0.24684 produces g(t) = 12 $e^{0.24684t}$, which furnishes the complete function model, even without the need for computation.

Table 2

1985	1990	1995	2000	2005	2010	2015
12	15	20	25	32	41	53

Table of approximate values for the function model g(t) discovered at t=2. Repeat with different values of t. Which is the more precise? To expedite the procedure, you are welcome to utilize a calculator and/or computer.

5 -The Problem Overview and Typical Example:

Following the talk and warm-up example, students are prepared for the real prototype issues. We will use this example to teach the pupils the fundamentals of the Leslie matrix. Let's start by examining a particular female population of a particular type of rabbit found in the Amazon Rainforest. Assume that there are only four age groups in this species of rabbit: Any rabbits produced during the current breeding season are classified as R0 = age 0. Compare the results of each model after including additional values (rounded to the nearest thousand) in the table. To illustrate, when t equals 2, the corresponding value of n = 0.24684 produces g(t) = $12e^{(0.24684t)}$, which furnishes the complete function model, even without the need for computation. R3 = age 3 refers to any 2-year-old rabbit that makes it through the end of the year and advances to the next phase.

We'll suppose that these four phases mark the conclusion of the life cycle for this type of rabbit. That means that all (or the majority of) the rabbits in age group 3 who survive will perish at the end of the year. You can overlook the small percentage that will survive since it won't have an impact on the overall figure. There is now a specific death rate and reproduction rate for every age group. Below, we will now go over this:

- R0 = Too young for this group. Thus, the rate of reproduction is zero.
- R1 = Assume that, on average, 1.2 female rabbits born in this age group will survive.
- R2 = Due to their youth, this generation will reproduce at the greatest rate. Assume that, on average, this age group produces 1.5 female rabbits that will make it through.

- R3 = Given that this is the oldest age group, we may infer that its rate of reproduction will be lowest. Assume that 0.7 female rabbits on average from this age group will survive.

In addition, we will assume the following for each age group's likelihood of surviving to the next age group:

- From t to t+1,
- 50% of the P0 population survives.
- R1 population survival from t to t+1 is 35%.
- 15% of the R2 population survives till t+1.
- Between time t and t+1, there is a complete absence of surviving individuals within the R3 demographic. (Please note that this statement assumes the conclusion of the age group at R3.)

Question no. 5: Have group discussions on whether the figures above make sense. Which are some real-world elements that might influence the calculation?

We will use a vector to represent the whole population at any one time:

$$\overrightarrow{R(t)} = \begin{pmatrix} R0(t) \\ R1(t) \\ R2(t) \\ R3(t) \end{pmatrix} \qquad \text{Equ}(1)$$

This shows the population for each age group at time "t." Our goal is to locate $\overrightarrow{R(t+1)}$ from $\overrightarrow{R(t)}$. Using all of the previously mentioned information, the following matrix multiplication will get this:

$$\overrightarrow{R(t+1)} = \begin{pmatrix} R0(t+1) \\ R1(t+1) \\ R2(t+1) \\ R3(t+1) \end{pmatrix} \qquad \text{Equ}(2)$$

Where,

$$\begin{pmatrix} R0(t+1) \\ R1(t+1) \\ R2(t+1) \\ R3(t+1) \end{pmatrix} = \begin{bmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{bmatrix}$$

put the above value in equation (2).

$$\overrightarrow{R(t+1)} = \begin{pmatrix} R0(t+1) \\ R1(t+1) \\ R2(t+1) \\ R3(t+1) \end{pmatrix} = \begin{bmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{bmatrix}$$

Solve equation(1) and equation (2)

$$\overrightarrow{R(t+1)} = \begin{pmatrix} R0(t+1) \\ R1(t+1) \\ R2(t+1) \\ R3(t+1) \end{pmatrix} = \begin{bmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{bmatrix} \times \begin{pmatrix} R0(t) \\ R1(t) \\ R2(t) \\ R3(t) \end{pmatrix}$$

Question no. 6: How does the provided data fit into the following matrix equation? Discuss as a group. The Leslie matrix is the name given to this four by four grid. A four-by-four Leslie matrix was derived from our analysis, as we solely considered four age categories.

Question no. 7: Let's consider that the variable "t" represents time in years, and the initial population is given in thousands.

$$\overrightarrow{R(0)} = \begin{pmatrix} R0(0) \\ R1(0) \\ R2(0) \\ R3(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

To determine the population after one and two years, use the matrix equation and the starting population.

6 Leslie Matrix Eigenvalues and Eigenvectors: Utilizing Mathematica for Analysis

Enter the Leslie matrix first, which we looked at in the earlier example.

$$\begin{bmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{bmatrix}$$

We utilize a collection of lists to input matrices in Mathematica. Thus, the matrix shown above is expressed as:

$$S = \begin{Bmatrix} \{0,1.2,1.5,0.7\}, \{0.5,0,0,0\} \\ \{0,0.35,0,0\}, \{0,0,0.15,0\} \end{Bmatrix}$$

Observe that this Leslie matrix is assigned the designation "S." To determine the collection of Eigenvalues and matching Eigenvectors, use the "Eigensystem" command.

Question no. 8: Firstly, establish that both O and P are eigenvectors of A, In the given context, A denotes a m by m matrix, a signifies a real eigenvalue, and the complex eigenvector is denoted as O + jP, where O signifies the real component and P signifies the imaginary component.

Question no. 9: In this physical context, the positive eigenvalues and their associated eigenvectors signify what? Conversely, what do the negative eigenvalues and their accompanying eigenvectors represent?

Question no. 10: Collaborative Appraisal: What distinguishes population modeling with a Leslie matrix from that with an exponential or polynomial function?

7- Examine the Long-Term Leslie Matrix Behavior:

Our objective is to investigate the long-term behaviour of the Leslie matrix, denoted as S.

$$\lim_{n\to\infty}(S)^n$$

To achieve this, we will utilize Mathematica's "Table" and "MatrixPower" commands. We'll use the following to list the first thirty powers of S:

Table [Matrix Power [*S*, *j*] / / Matrix Form, {*j*,1, 25}]

Question no. 11: Talking in Groups – Examine the first 25 Leslie matrix powers. Do you see any restrictive actions? Make an effort to compute and analyze a few more power.

The computation of the power of a matrix when it is applied to a specific initial vector is also possible. In other words, given an initial population and a Leslie matrix, our goal will be to identify the long-term population that approaches the original population (if such a population exists). Return to at this point and apply the identical starting condition that is specified there. All we do is execute the following command:

Power [Matrix S,10, {2, 3, 4, 3}]

After ten years, this will provide the population.

Question no. 12: To determine the population after 15, 20, and 50 years, use the aforementioned program. What inference do you make from this? If the starting population is, what will the population's long-term behavior be $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$?

Actually, Mathematica has additional capabilities. The command is indeed usable here:

Matrix Power [*S*, *n*]

This will compute the n^{th} power of S symbolically. The limiting behavior may now be found using the "Limit" command:

Limit [Matrix Power [*S*, *n*], *n* → Infinity]

Question no. 13: What response did the preceding command provide you?

3.5. The undertaking

Question no. 14: Given the previous discussion and the technique used to build a Leslie matrix, what is the overall structure of a Leslie matrix? List some common examples of both a 6 by 6 and a 5 by 5 Leslie matrix.

Question no. 15: Suppose we have the following Leslie matrix for a species belonging to the age group six.

$$\begin{bmatrix} 0 & 1.2 & 1.5 & 2 & 1.2 & 0.02 \\ 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$$

Examine the Leslie matrix here. Complete all of the computations using our 4-by-4 Leslie matrix example. Think about several starting populations, such as the following:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$$

Question no. 16: What limitations accompany the utilization of a Leslie-type matrix in population modeling?

Question no. 17: The "female" population is often studied using the Leslie matrix. Do you know why this is happening specifically?

Question no. 18: Describe an instance in which there are eight age groups. Create and evaluate the Leslie matrix.

Question no. 19: If you want to simulate the human population using the Leslie matrix, how many age groups will there be? Do you believe that the human population will respond well to this model? Why not, if not?

8. CONCLUSION:

As outlined in the introductory section of the article, the final project holds the dual function of not only serving as a culmination of the students' efforts but also functioning as an assessment tool to measure their attainment of learning objectives. Its adaptability allows for seamless adjustments, modifications, and variations of the module, catering to the diverse needs and preferences of both educators and learners alike.

REFERENCES:

[1] F. Gewers, G. R. Ferreira, H. F. Arruda, F. N. Silva, C. H. Comin, D. R. Amancio, and L. da F. Costa. Principal component analysis: A natural approach to data exploration. Researchgate, 2019.

[2] R. A. Horn and C. R. Johnson. Matrix Analysis.Cambridge University Press, 2012.

[3] S. Gershgorin. ¨Uber die abgrenzung der eigenwerte einer matrix. Izv. Akad. Nauk. URSS Otd. Fiz.-Mat.

[4] Wikipedia. Eigenvalues and eigenvectors. Wikipedia, the free encyclopedia, 2020.

[5] Ormrod, J. E. (2003). Educational Psychology: Developing Learners. Merrill Prentice Hall.

[6] H. Sagan. Boundary and eigenvalue problems in mathematical Physics. Dover, 1989.

[7] Engle, J., & Tinto, V. (2008). Moving beyond access: College success for low-income, first generation students. *Pell Institue for the Study of Opportunity in Higher Education*. Retrieved from http://www.eric.ed.gov/PDFS/ED504448.pdf.

[8] L. da F. Costa. Features transformation and normalization. Researchgate, 2019.

[9] Susan, M., & Linda, N. (1998). Student learning styles and their implications for teaching. *Center for research on Learning and Teaching (Occasional Papers)*(10).

[10] Liu X. A new method for calculating derivatives of eigenvalues and eigenvectors for discrete structural systems. *Journal of Sound and Vibration*. 2013;332(7):1859-1867. doi:10.1016/j.jsv.2012.11.017