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Axiom Countability of via Semi- open sets

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Abstract: The paper intends to present a s-open sets and discuss some of its properties. and its relationship to open sets. also, they are used to introduce a specific form of countable space axioms called s-countable axioms, these include; S- separable spaces, s-first countability and s-second countability. Through the classical concept of topology, the properties of these spaces have been studied; moreover, we study the enumerated s-compounds and their behavior in some special spaces.

Keywords: Topological spaces and generalization, s- open sets, subspace, Countability Axiom Separability.

Introduction

Introduction

Many researchers have introduced different types of generalized countability axioms:

g-countability axiom, b-countability axiom, D-countability axiom and finally R-countability axiom. The cared Siwiec in a year 1974 [1] The g-(first, second) countable space are defined by

using weak basis in (X,τ) space, and showed their relationship with measurability. A year later, Siwiec [2] wrote an overview that generalized the concept of first countable space and studied the relationship between these generalizations.

In the year1991, Jian-ping [3] studied some of them Generalization of the first countability;

Call it namely ω_k- spaces, when he states

The relationship between the T_1 spaces , ω-

spaces and first countability. In 2013, Selvarani [4] advance guard the b-countable axiom on b-open sets, and then Elbhilil and Arwini[5] defined generalize types of countability axiom. is called pre-first countability, and they define these axioms in terms of sets, proving that pre-separable spaces and pre-second countability are equivalent to separable spacesIn general topology, several distinct understandings of open sets have been

explored.

Some of these concepts, including i-open set

(i=α,s,pre,b)

,have been defined in a similar manner using operations involving the closure and the interior. Of these, the notion of preopen (or locally dense) sets is particularly significant. "Locally dense" sets were first identified as preopen sets by Corson and Michael in 1964 [6]. In 1982, Mashhour, and others [7] used the term "preopen" instead of "locally dense" set. Introduced by Csaszar [8], a set X can have a generalized topology - abbreviated as GT - which is typically represented as (X,μ), in this terminology, the μ-open sets indicate the elements of the generalized topology. The concept of μ-countability

axioms, which are specific to GT, were established in 2013 by Ayawan and Canoy [9]. In their research, they explored the qualities of these ideas and determined the characteristics of μ-first (μ-second) countable space in the context of GT product. See [10,11] for details on fundamental properties in generalized topology. In 1963, Levine [12] took an interest in the concept of s-open sets.
The study of general closed sets was started by C.E.Aull [13] in 1968, We consider the set of closed sets that belong to each superset as open. Arya and Noor introduced the concept of generalized closed set. [14] In 1987, Bhattacharyya and Lahiri defined and tested the concept of s-generalized closed sets based on the concept of s-closed sets. This class was introduced of α -generalized closed sets by Maki, Devi and Balachandran [15] in 1994.
In this study we used s-open sets to define a countability axiom, it is called s-countability axioms, where this class consists of the axioms: s-separability, s-first countability and s-second countability.
It is illustrate the relationship between the countable axioms and the s-countability axioms, then we examine the hereditary
properties of these spaces and their images under special functions, and the properties of these spaces: submaximal spaces, partitioned spaces, extreme separation spaces, solvability.
The article is divided into seven main parts: s-
open sets, s-dense, s-continuous functions, s-
separability spaces,s-first countability, s-
second countability, properties of s-
countability, and final state.

2. SEMI OPEN SETS

Definition 2.1. [16] A subset η of space X it's say s-open if and only if there exists u an open set such that $u \sqsubseteq \eta \sqsubseteq \bar{u}$.

Theorem 2.1. [16] A subset η in X space is s-open if and only if $\eta \sqsubseteq \overline{(\eta^O)}$.

The all of s-open sets and s- closed sets in (X,τ)are denoted by SO(X) and SC(X),
respectively.

Theorem 2.2. [12] Let η be s-open in the space X and suppose $\subseteq \xi \subseteq \bar{\eta}$. Then ξ is s-
open.

Definition 2.2. [16] The s-closed set is complement of a s-open set.

Proposition 2.1. [17]

X and ϕ are s-open sets.

Arbitrary union of s-open sets is s-
open.

Intersection of s-open sets need not be s-open.

Examples 2.1.

If X=R with usual topology space and let

A={x: 0<x<3}∪{4} , B={x: 2<x<5}∪{1}

A and B are not s-open, but A∪B are s-open set.

2) Let ={a ,b,c} , τ={X ,ϕ ,{a}} is topology space then {b} is s–open but is not open set.

Definition 2.3. [18] The intersection of all s-closed sets of X containing η is called the s-closure of η and is denoted by $\bar{\eta}^{(semi)}$.

Definition 2.4. [18] The union of all s-open of sets of X contained in η is called the s-interior of η and is denoted by $(\eta^o)^{semi}$.

Proposition 2.2. [19] A s-open set is the intersection of an open set and a s-open set.
Definition 2.5. [6] A subset P of a space (X,τ) is say preopen if $P \subseteq \overline{P}^{O}$, the preclosed set is complement of preopen set.
In [6] the following diagram (1) shows the relationship between the open, peropen set and dense set.

open ⇒ preopen
Dense⇒ preopen

Diagram (1)

Proposition 2.3. A set η in a space(X,τ) then
1) In [20] $\eta \subset \overline{\eta}^{(semi)} \subset \overline{\eta}$.
2) If η is s-open [21] in X and P is preopen in X such that $\eta \cap P = \phi$, then $\eta = \phi$ or $P = \phi$.
Theorem 2.3. [12] Let $\eta \subseteq Y \subseteq X$ where X is a space and Y is a subspace. Let $\eta \in SO(X)$. Then $\eta \in SO(Y)$.
Definition 2.6. [22] A set is regular open if and only if it is preopen and s-closed.
Theorem 2.4. [22] For a subset η of (X,τ) the following sentences are equivalent:
1) η is preopen.
2) The s-closure of η is a regular open set.
Definition 2.7. [23] A subset η of a space X it is said to be s-regular if a subset η is s-open and s-closed .
The set of all s-regular sets of X is denoted by SR (X).
The following diagram (2) shows the relationship between the regular sets and s-open sets.

regular sets ⇒s-regular sets ⇒ s-open sets

Diagram (2)

Proposition 2.4. [24] If (X,τ) is topological space, and $\eta \subset P \subset X$, where P is preopen. Then η is s-open (s-closed) in P if and only if $\eta = S \cap P$, for some s-open (s-closed) set S.
Proposition 2.5. [25] Let η be a subset of a space X. Then:
$\overline{\eta}^{semi} = \eta \cup \overline{\eta}^{O}$.
2) $\overline{(X-\eta)}^{semi} = X - (\eta^{O})^{semi}$.
3) $((X-\eta)^{O})^{semi} = X - \overline{\eta}^{semi}$.
Definition 2.8. [26] A subset B of a space X is say b-open if $B \subseteq \overline{(B^{O})} \cup \overline{B}^{O}$.
Theorem 2.5. if (Y,τ_Y) be a subspace of a space (X,τ) and let $\eta \subseteq Y$. If η is s–open in X , then η is s -open in Y.
Proof. Let η is s-open in , there exists an open set u in X such that $u \subseteq \eta \subseteq \overline{u}$. Then $u^{Y} = u \cap Y \subseteq \eta \subseteq \overline{u} \cap Y = \overline{u}^{Y}$ Thus η is s-open in Y.
Definition 2.9. [27] A subset w of a space X is Called a weak open, if there is an open set u such that $\overline{w} = \overline{u}$.
Remark 2.1. 1) IN [27] All s-open sets are weakly open set.
A [6] set is an open if and only if it is s-open)and preopen.
The following diagram (3) shows the relationship between the open sets and i-open sets (i=α ,pre ,s ,b)

Open set ⇒ α-open set ⇒ (⇒s-open set / ⇒pre open set) ⇒ b- open set

Diagram (3)

Proposition 2.6. [6] If η is s-open in space X and P is preopen in space X, then $\eta\cap P$ is s-open in P and preopen in η.

3. S-DENSE SET.

Definition 3.1. [28] let D a subset of X a space is said to be dense if $\overline{D}=X$.

Definition 3.2. [29] A subset B of X is called b-dense if $\overline{B}^{b}=X$.

Definition 3.3. [30] let D a subset of X a space is said s-dense if $\overline{D}^{semi}=X$.

Proposition 3.1. [20] Every non-empty preopen subset of X space is s-dense.

Corollary 3.1. let (X,τ) topology space then:

1) Any s-dense set is dense.

2) Any b -dense set is s-dense.

Proof. 1) if D be a s-dense subset of a space X, i.e. $\overline{D}^{semi}=X$, since $\overline{D}^{semi}\subseteq\overline{D}$ then $\overline{D}=X$.

2)if B be a b-dense subset , i.e. $\overline{B}^{b}=X$, since $\overline{B}^{b}\subseteq\overline{B}^{semi}$ such as $\overline{B}^{semi}=X$.

The following diagram (4) shows the relationship between the b–Dense and s-open sets.

b-Dense ⟹ s-Dense ⟹Dense⟹ preopen⟹

s-open

Diagram (4)

Corollary 3.2. If A subset of (X,τ) is s-dense if and only if any non-empty s-open set in contains points from A.

Proof. ⟹ If A be a s-dense subset in , and if Bbe a non-empty s-open set. Since $B\neq\phi$ and there is $\overline{A}^{semi}=X$ there is $x\in B$ and $x\in\overline{A}^{semi}$,we get $A\cap B=\phi$.

⟸ If x be any element in X ,then any s-open set that contains x intersect A , i.e. $x\in\overline{A}^{semi}$ we get $\overline{A}^{semi}=X$.

Corollary 3.3. Let (X,τ) topology space then:

1)Every subset of X space that contains a s- dense set is s -dense.

2) If A is s-dense set in B,and B is s-dense in X, then A is s-dense in X.

Proof. 1) verified $\overline{A}^{semi}\subseteq\overline{B}^{semi}$for every sets A and B satisfy $A\subseteq B$.

2) Let N is s-open set in X,then $A\cap B\neq\phi$ since B is s-dense in X ,by corollary (3.2) we have $N\cap B$ is s-open set in B. Since A is s-dense in , then we have $(N\cap A)\cap B\neq\phi$,i.e. $(N\cap B)\cap A=N\cap A\neq\phi$,Therefore, A is s-dense in X.

Theorem 3.1. [31] A set $\eta\subseteq X$ is nowhere s-dense if and only if $(\overline{\eta}^{semi})^{O}=\phi$.

Theorem 3.2. [30] Let (X,τ) be a space and $D\subset X$.Then D is dense in X if and only if $D\cap\eta\neq\phi$.

For any non-empty $\eta\in SO(X)$

Proposition 3.3. [30] Let (X,τ) be a space and $\in SO(X)$, $U\in\tau$ and $U\cap\eta\neq\phi$. Then for dense set , $U\cap\eta\cap D\neq\phi$.

Remark 3.1.[30] If D is dense in the space (X,τ) and U is open in (X,τ) then $\overline{((D\cap U))}$ is a s-open set.

Theorem 3.3. The union of finite number of non- s-dense set is non s-dense sets.

Proof. Let A,B are non-s–dense sets, we put $W=((\overline{(A\cup B)})^{semi})^{o}$ so that

$W\cap((\overline{B})^{semi})^{C}\subseteq((\overline{A})^{semi}\cup(\overline{B})^{semi})\cap((\overline{B})^{semi})^{C}$

that is $W\cap((\overline{B})^{semi})^{C}\subseteq((\overline{A})^{semi}\cup(\overline{B})^{semi})\cap((\overline{B})^{semi})^{C}$

〖A ¯ 〗^semi∩ (〖〖B ¯ 〗^semi)〗^C ⊆ 〖A ¯ 〗^semi. Since 〖B ¯ 〗^semi∩(〖〖B ¯ 〗^semi)〗^C=ϕ.then 〖〖(W∩(〖〖B ¯ 〗^semi)〗^C)〗^O〗^semi⊆ (〖〖B ¯ 〗^semi)〗^C⊆〖(scl (A))^O〗^semi=ϕ ⊆ 〖∅^O〗^semi. Since A is non s-dense. But

(〖〖W∩ (〖〖B¯〗^semi)〗^C)〗^O〗^semi=W∩ (〖〖B¯〗^semi)〗^C.

Since W∩(〖〖B¯ 〗^semi)〗^C is s–open set, it follows that W∩(〖〖B¯ 〗^semi)〗^C=ϕ, which implies

W⊆ 〖B¯〗^semi then 〖w^o 〗^semi⊆ 〖(〖B¯〗^semi)^o〗^semi=ϕ, Since B is non s–dense. But 〖w^o 〗^semi= 〖〖〖〖〖(A∪B) ¯ 〗^semi〗^o〗^(semi)〗^o〗^semi=〖〖〖(A∪B) ¯ 〗^semi〗^o〗^semi. So that

〖〖〖(A∪B)¯〗^semi〗^o〗^semi=ϕ .Hense A∪B is non s- dense.

Theorem 3.4. If A be a subset of(X,τ) spaces
then A is non s– dense in X if and only if
X-〖A¯〗^semi is s- dense in X.
〖A^o 〗^semi=X-〖(X-A)¯〗^semi

Proof. by Proposition 2.6. it follows that 〖A¯〗^semi=X-〖(X-A)^o 〗^semi and
〖〖〖A¯〗^semi〗^o〗^semi=X-(X-〖A¯〗^semi)¯^(semi)
Sinse A is non s–dense then 〖〖〖A¯〗^semi〗^o〗^semi=ϕ.
then X-(X-〖A¯ 〗^semi)¯^(semi)=ϕ, we get (X-〖A¯ 〗^semi)¯^(semi)=X,then (X-〖A¯ 〗^semi)¯^(semi)is s–dense.

4.S-CONTINUOUS FUNCTION.

Definition 4.1. [12] let F:(X,τ)→(Y,σ) function is called s-continuous if is F^(-1) (V) a s-open set of (X,τ) for any open set V of (Y,σ) .

Corollary 4.1. Any continuous function is s– continuous function.

Theorem 4.1. [12] Let F:(X,τ)→(X,σ) be the s-continuous function. Then for any dense
subset D of (X,τ) , D∩F^(-1) (O)≠ϕ, for anyO∈σ.

Definition 4.2. [33] A function F:X→Y is said to be:
1)irresolute if the inverse image of every s-open set in Y is s-open in X.
2)pre-s-open if the image of every s-open set in X is s-open in Y.

Definition 4.3. [33] if F:X→Y function is said to be s-homeomorphism if F is irresolute
and pre-s-open.

Theorem 4.2. [33] If F:X→Y then:
1)An open and continuous then F is irresolute and pre s-open.
2) a homeomorphism then Fis also a s- homeomorphism.

Theorem 4.3. [34] Let A is s–open set in X space, if F:X→Y be a continuous open mapping where X and Y are spaces . then F(A) is s-open set in Y.

5. APPLECATION OF SEMI-OPEN SET

5.1.In Partition Spaces.

Definition 5.1.1. [28] A (X,τ) be a space is say partition space if any open subset of X is
closed.

Proposition 5.1.1. In partition space (X,τ) , any subset of X is s-open, i.e SO(X,τ)=P(X).

Proof. Suppose A is a subset of X ,then A⊆A¯, every closed set in partition space is also open, then A^- is open set, i.e. $(A^O)^- = A^-$, so $A \subseteq (A^O)^-$, then A is s-open set.

Corollary 5.1.1 In Partition space X is s-separable space if and only if the space X is countable space.

Proof. clear that $\{\{x\}\}$ is a countable s-local base at every point x in X .

Definition 5.1.2. [39] A topological space (X,τ) is say s-partition if any s-open subset of X is s-closed.

5.2.In Submaximal Spaces.

Definition 5.2.1. [35] Let (X,τ) is Say submaximal space if each dense set in X is open.

Theorem 5.2.1. [36] If (X,τ) is submaximal and A∈SO(X) then (A,τ_A) is submaximal.

Corollary 5.2.1. In (X,τ) is submaximal, any s-open set is open, i.e. SO(X,τ)=τ.

Proof. If A be a s-open subset in a space , then from proposition (2.4) (1) A is preopen set in X in [5] we get A is open set.

Corollary 5.2.2. In a submaximal space X, if D is dense if and only if D is s-dense in X.

Definition 5.2.2. A space (X,τ) is called s-submaximal if any s-dense subset of X is s-open.

Definition 5.2.3. [37] A subset η of a space (X,τ)is say s-preopen if $\eta \subseteq ((\eta)^{-o})^-$, s-preclosed set is the complement of s-preopen set .The family of all s-preopen sets and s-preclosed sets in X are denoted by SO(X) and SC(X),respectively.

Theorem 5.2.2. If (X,τ) be a space, then the following properties are holds:

(X,τ) is s-submaximal;

2)any s-preopen set is s-open.

Proof. 1) ⟹ 2): if $N \subseteq X$ be a s-preopen set. Then $\subseteq (N^O)^{-O}$, let $(N^O)^{-O}=M$, say.

This implies $(M)^{-semi}=(N)^{-semi}$ and hence $((X-M)\cup N)^{-semi}=((X-M))^{-semi}\cup(N)^{-semi}=((X-M))^{-semi}\cup(M)^{-semi}=X$ and thus $(X-M)\cup N$ is s-dense in .

$N=((X-M)\cup N)\cap M$ is s-open.

Now

$(X-M)\cup N$

and N is the intersection of two s-open sets and hence N is s-open.

⟹(1 : Let M be a s-dense subset of X.

Then $((M)^{-semi})^o=X$, then $M\subseteq ((M)^{-semi})^O$ and M is s-preopen,M is s-open.

5.3.In extremely disconnected

Definition 5.3.1. [43] A space (X ,τ) is called extremelly discometed if the closure of evrey coopen set is open.

Theorem 5.3.1. [20] In a topological (X,τ) the following nditions are equivalent:

X is extremally disconnected.

2)Any regular closed subset of X is preopen.

3)Any s-open subset of X is preopen.

Corollary 5.3.1. [21] A topological (X,τ) is said to be extremally disconnected if and only if for any s-open set A⊆X and every s-preopen set ⊆X , then set A∩B is s-open.

Definition 5.3.2. A space (X,τ) is called extremally s-disconnected if the s–closure of any s-open subset of X is s-preopen.

Definition 5.3.3. Let subset N of a space (X,τ) is said to be regular s-open if $N=((N^{-semi})^o)^{semi}$.

The complement of a regular s-open set is called regular s-closed.

Proposition 5.3.1. Let (X,τ)be a topological space and $N \subseteq X$.If N is a regular s-open set, then it is s-open.

Proof. Clearly.

Theorem 5.3.2. Let (X,τ) be a space, then the following are equivalent:

1)(X,τ) is extremally s-disconnected;

2)Any regular s-open set is s-clopen.

Proof. 1) ⇒2): Let (X,τ) is extremally s-disconnected and Let N be a regular s-open set, then $N=(((\overline{N}^{semi})^{o})^{semi}=\overline{N}^{semi}$ Hence N is s-closed ,we have N is s-clopen.

⇒2) : Let $N \in SO(X)$. Then $\overline{(N^{O})}$is a regular s-closed set which is s-clopen. Hence is $\overline{(N^{O})}$s-open.

5.4.On Resolvability

Definition 5.4.1. [40] A space (X ,τ) is said to be resolvable if there is dense subset D-X for which X-D is also dense. A space which is not resolvable is called irresolvable. Corollary 5.4.1. [40] Every subset of X is resolvable (irresolvable)if it is resolvable (irresolvable) as a subspace.

Corollary 5.4.2. [41] Any submaximal space is irresolvable and in fact hereditarily irresolvable.

Theorem 5.4.1. [41] Any s-open subset of a resolvable space is resolvable.

6.S-SEPARABILITY.

Definition 6.1. [29] A topological (X ,τ) is said to be separability if it has a countable dense subset in X.

Theorem 6.1. [29]

1)Every open subspace of a separability is separability.

2) The continuous image of a separability space is separability.

Definition 6.2. [30] A space X is said to be b-separability space if it has a countable b-dense subset of X.

Definition 6.3. A space X is say s-separability if it has a countable s-dense subset of X.

Corollary 6.1. 1) Any s-separability is separability.

2)Any b-separability space is s-separability.

Theorem 6.2. Every s–open subspace of s-separability is s-separability.

Proof. Let Y is a s-open subspace of s-separability X , then X has a countable s-dense subset η , since Y is a s-open subspace, then $Y \cap \eta$ is s-dense and countable subset in Y ,hence Y is s-separability.

Remark 6.1. An open subspace of s-separability is s-separability.

Proof. Direct since any open set is s-open. Theorem 6.3. A s-irresolute image of s-separability is s-separability.

Proof. Let $F:X \to Y$ be a s-irresolute function from a s-separable.

Be a s-irresolute function from a s-separability space , then X has a countable s-dense subset η , so $F(\eta)$ is countable. Now suppose N is a s-open set in $F(X)$ since Fis s-irresolute is a non-empty s-open set in X, so $F^{-1}(N) \cap \eta \neq \emptyset$,hence $N \cap F(\eta) \neq \emptyset$. We have $F(\eta)$ $F^{-1}(N)$ is a countable s-dense subset of F(X).

7.S-FIRST COUNTABILITY

Definition 7.1. [28] A space X has a countable basis at x if there is a countable collection B_x of neighborhoods contains x is say basis at X if for each neighborhood U such that $x \in U$ there exists B_x in B_x such that

$x \in B_x \subseteq U$.

Definition 7.2. [28] A topological space X having a countable basis at each of its points is said to first countability if for any $x \in X$ there is a countable local base B_x at X.

Theorem 7.1. [28] 1) Every subspace of first countability is first countability.

2) The continuous image of a first countability and open map is first countability.

Definition 7.3. In a space X, a collection N_x of s-open sets that contains an element x is called s-local basis at X if for any s-open set B such that $x \in B$ there is $N_x in N_x$ such that $x \in N_x \subseteq B$.

Definition 7.4. A topological space X is called s-first countability if for any $x \in X$ there is a countable s-local base at X.

Theorem 7.2. A s-open subspace of s-first countability is s-first countability.

Proof. Let Y be a s-open subspace of a s-first countability X, then any $y \in Y$ ($\subseteq X$) has a countable s-local base $\aleph_y$ for X.

Let M is a s-open set in Y that contains y, then form theorem (2.6)then M is s-open in X , since $\aleph_y$ is a s-local base at y , there exists a s-open set $〚N_y \in \aleph〛_y$ such that $y \in N_y \subseteq M$, then $y \in N_y \cap Y \subseteq M \cap Y = M$therefore $N_y \cap Y$ is a countable s-local base at y in the subspace Y.

Theorem 7.3. Image of s-first countability under s-irresolute and M-s-open map is s-first countabililty.

Proof. Suppose F: X⟶ Y is a s-irresolute, M-s-open map from a s-first countabilily X onto a space Y. Then for any $y \in F(X)$ there is a countable s-local base $\aleph_{(F^{-1}(y))}$ at $F^{-1}(y)$ for , since F is M-s-open map the collection $F(\aleph_{(F^{-1}(y))})$ is countable collection of s-open sets in F(X) , and since F is s-irresolute, then $F(\aleph_{(F^{-1}(y))})$ is a countable s-local base at y .

8. S-SECOND COUNTABILITY

Definition 8.1. [29] A space X has a countable basis if there is a countable collection B of subsets of X that is a basis for on X space. In this case X is said to satisfy the second countability axiom, or to be second-countable.

Theorem 8.1. [29]

(1Second-countability implies first countability.

2) Every subspace of a second-countability is second-countability.

3) The continuous image of a second countability and open map is second countability.

Definition 8.2. The collection $\aleph$ of s-open sets in a space (X,τ) is say s-base for if each s-open set can expressed as a union of members of $\aleph$.

Examples 8.1.

1)Let X=R with $\tau=\{R,Q,K,\emptyset\}$ the collection {Q ,K } is base for R but not s-base, while the collection is $\{\{x\}\}_{(x \in R)}$s-base for R but not base.

2)Let $\tau=\{R,\emptyset\}$ on R the collection $\{\{x\}\}_{(x \in R)}$ is s-base but not base.

Definition 8.3. A space (X,τ) is say s- second countability if X has a countable s-base.

Examples 8.2.

(1Let X=R with $\tau=\{R,Q,K,\emptyset\}$ is second countability but not s-second countability.

(2The trivial space on uncountable is second countable space but not s-second countability.

(3The space X=R with $\tau=\{R,Q,\emptyset\}$ is s-second countability, since $\{R\} \cup \{\{x\}\}_{(x \in Q)}$ is a s-local base for R.

Definition 8.4. Let (X,τ) be a space is say s-countabiity if the collection $SO(X,\tau)$ is countability.

Corollary 8.1. Any s-countability is s-second countability.
Proof. Direct since SO(X ,τ) is a countability s-local base for X.
Remark 8.1. Any s-countable space is s-first countability space and s-separability.
Examples 8.3.
(1Let X=R with τ={ϕ}∪{A⊆R:1∈A } is s-separability(since {1} is countable s-dense set in R), but X is not s-second countability since {1,x} is s-open set for any x ∈R, x≠1.
(2If X=R and τ={ϕ,R ,{1}} then X is s-first countability (since {{1,x}} is a countable s-local base at any point x∈R), but not s-second countability. Note that, the space X is countable but not s-second countability.
Theorem 8.2. Any s-open subspace of s-second countability is second countability.
Proof. If Y be a s-open subspace of a s-second countability of X, then X has a countable s-base ℵ. Now we need to prove that the collection $ℵ_Y$={N∩Y:N∈ℵ} is a countable s-base for Y Let M is a s-open set in Y , then form theorem (2.3) we get M is s-open in X, since ℵ is a s-base for X, there exist s-open sets $N_α$∈ℵ such that M=∪$N_α$, then M=M∩Y=∪($N_α$∩Y) , therefore {$N_α$∩Y} is a countable s-base for the subspace Y.
Theorem 8.3. The s-irresolute image of s-second countability and M–s-open map is
s-second countability.
Proof. Let F:X→Y is a s-irresolute and M-s-open map from a s-second countability X onto space Y . Then X has a countable s-base ℵ , since the map F is M- s-open map the collection F(ℵ) is a countable collection of s-open sets in F(X) , and since F is s-irresolute, then F(ℵ) is a countable s-local base for F(X).
Example 8.4. Let X=Y=R with $τ_1$={R,Q,ϕ} then SO(X,$τ_1$)=P(Q)∪R,and $τ_2$={R,Q,ϕ} ,
then SO(X,$τ_2$)=P(R). Then the identity map from (R,$τ_1$)onto the space (R,$τ_2$) is M-s-open map, however the space (R,$τ_1$) is s-second countability while (R,$τ_2$) is not s-second countability.

Conclusion

We introduce the notion of dense countability axioms; namely semi-countability axioms. We study the basic properties of s-countability axioms, as their subspaces and their continuous images. In addition, we discuss the relations between s-countability axioms and countability axioms, and we prove that the axioms of separability, s-separability and s-second countability are equivalent.

Outline some of our results:
s-dense set is dense.
In submaximal space, any s- open set is open set.
In partition space, any subset is s-open.
s-separability is separability, but not conversely.
b-separability is s-separability space, but not conversely.
A s-open subspace of s-first countability is s–first. countability.
s-countability space is s-first countability and s-separable space.
s-Second Countabililty is s-First Countability.

- s-open subspace of s-second countability is second countability.

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