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Explicit Optical Solutions for the Nonlinear Chen-Lee-Liu Equation Utilizing the Exp-function Scheme

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الحلول البصرية الصريحة لمعادلة تشن – لي – ليو غير الخطية باستخدام طريقة الدالة الأسية

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الملخص:

في هذه الورقة سنستخدم طريقة الدالة الأسية بمساعدة برنامج ميبل لإنشاء حلول بصرية صريحة **لمعادلة تشن - لي – ليو**.
طريقة الدالة الأسية هي واحدة من أكثر الطرق التحليلية التي تستخدم لحل أنواع متعددة من المعادلات التفاضلية الجزئية غير الخطية في الفيزياء الرياضية.
هذه الطريقة تعتبر واضحة ومختصرة وفعالة وأداة رياضية قوية للبحث عن الحلول البصرية الصريحة للمعادلات التفاضلية الجزئية غير الخطية في الفيزياء الرياضية.

الكلمات الدالة: حلول بصرية صريحة، معادلات تفاضلية جزئية غير خطية، معادلات تشن – لي – ليو غير الخطية طريقة الدالة الأسية.

Abstract

This paper will utilize the Exp-function scheme with the aid of Maple software to construct new explicit optical solutions to the nonlinear Chen-Lee-Liu equation.
The Exp-function scheme is one of the most recent analytical schemes which used to solve many types of nonlinear PDEs in mathematical physics.
This scheme considers a straightforward, concise, effective, and powerful mathematical tool to look for new explicit optical solutions to nonlinear PDEs in mathematical physics.

Keywords: Explicit optical solutions, nonlinear PDEs, nonlinear Chen-Lee-Liu equation, the Exp-function scheme.

1. Introduction

In the recent years, many new natural phenomena exist in mathematical physics and some other fields as optical fibers, plasma physics, solid state physics, biology, chemistry, engineering, chemical kinematics, quantum mechanics, fluid mechanics, hydrodynamic waves and etc., which can be describe by nonlinear PDEs [1-18].

There are many analytical schemes to obtain explicit optical solutions for the nonlinear PDEs in mathematics, physics, engineering and some other fields such as the modified extended tanh-function scheme [1-5, 17], the generalized $\left(\frac{G'}{G},\frac{1}{G}\right)$ expansion scheme [10], the Exp-function scheme [11], the Jacobi-elliptic function scheme [12], the auxiliary equation scheme [13], the generalized-projective Riccati equations scheme [14], the modified algebraic scheme, the new mapping scheme, etc.

The Exp-function method, first introduced by He and Wu in 2006, has become a valuable tool for finding analytical solutions to a wide range of nonlinear partial differential equations [11].

This article aims to apply the Exp-function method [5, 6, 18], supported by Maple software, to derive new explicit optical solutions for the nonlinear Chen–Lee–Liu equation [1].

$$\Psi_t + a\Psi_{zz} + i(\vartheta^2\beta+\alpha)|\Psi|^2\Psi_z - i[\lambda\Psi_z + \theta(|\Psi|^2\Psi)_z + \mu(|\Psi|^2)_z\Psi] = 0, \qquad (1.1)$$

Here, $\Psi(z,t)$ is a complex-valued wave profile with $i=\sqrt{-1}$. The first term denotes linear temporal evolution, while $a,\ \vartheta^2\beta+\alpha$ are respectively, the coefficients of chromatic dispersion and nonlinear dispersion. The parameters $\lambda,\theta,$ and μ correspond to intermodal dispersion, self-steepening, and nonlinear dispersion, respectively.

The structure of this article is as follows: Section 2 introduces the Exp-function method, while Section 3 presents the auxiliary formulas required for its implementation. In Section 4, the method is applied to the nonlinear Chen–Lee–Liu equation. Finally, Section 5 summarizes the main conclusions.

1. Description of the Exp-function scheme

Consider a nonlinear partial differential equation for $\Psi(z,t)$ of the form:

$$P(\Psi,\Psi_z,\Psi_t,\ \Psi_{zz},\Psi_{zt},\Psi_{tt},\dots)=0, \qquad (2.1)$$

Where P is a polynomial in its arguments. The core idea of the Exp-function method can be outlined through the following steps:

Step 1: To obtain traveling wave solutions of Eq. (2.1), we assume

$$\Psi(z,t) = \Psi(\zeta);\ \ \zeta = z-\vartheta t\,, \qquad (2.2)$$

where ϑ is a constant, z is the independent variable and t is a parameter. Therefore, the transformation (2.2) converts Eq. (2.1) to the nonlinear ODE as follows:

$$Q(\Psi,\Psi',-\vartheta\Psi',\Psi'',\vartheta^2\Psi'',\dots)=0, \qquad (2.3)$$

where prime denotes the derivative with respect to ζ.

Step 2: We assume that Eq. (2.3) has the formula solution:

$$\Psi(\zeta)=\frac{\sum_{i=-c}^{p}\ w_i\,e^{i\zeta}}{\sum_{j=-d}^{q}\ s_j\,e^{j\zeta}}=\frac{w_{-c}\exp(-c\zeta)\ +\cdots+w_p\exp(p\zeta)}{s_{-d}\exp(-d\zeta)\ +\cdots+s_q\exp(q\zeta)}\ , \qquad (2.4)$$

The positive integers c,d,p and q are free parameters, meaning they can be chosen arbitrarily. The values of the constants w_i and s_j remain unspecified.

Step 3. Substituting Eq. (2.4) into Eq. (2.3).

Step 4: To find the values of c and d, we look at the simplest (lowest-order) linear term in Eq. (2.3) and match its behavior with the simplest nonlinear term. Basically, we make sure they "balance" each other in terms of how they scale or contribute. In the same spirit, we figure out p and q by doing a similar balancing act, but this time with the most complex (highest-order) linear term and the most complex nonlinear term. It's like tuning both ends of the equation so everything fits together smoothly.

Step 5. We are collecting terms of the same term of each power of $\exp(i\zeta)$ $(i = 0, \pm 1, \pm 2, \pm 3, \ldots)$.

Step 6. We equate the coefficients of each power of $\exp(i\zeta)$ to zero.

Step 7. We solve a set of algebraic equations to determine the unknown constants.

Step 8. Substituting the Solutions of Step 7, into Eq. (2.4) we have the exact wave solution.

3. Auxiliary formulas for the Exp-function scheme

On using the ansatz given by Eq. (2.4), we obtain general formulas as follows:

$$\Psi'(\zeta) = \frac{\tau_1 \exp[-(c+d)\zeta] + \cdots + \sigma_1 \exp[(p+q)\zeta]}{\varrho_1 \exp[(-2d)\zeta] + \cdots + \Gamma_1 \exp[(2q)\zeta]} \tag{3.1}$$

$$\Psi''(\zeta) = \frac{\tau_2 \exp[-(c+3d)\zeta] + \cdots + \sigma_2 \exp[(p+3q)\zeta]}{\varrho_2 \exp[(-4d)\zeta] + \cdots + \Gamma_2 \exp[(4q)\zeta]'} \tag{3.2}$$

$$\Psi'''(\zeta) = \frac{\tau_3 \exp[-(c+7d)\zeta] + \cdots \ldots + \sigma_3 \exp[(p+7q)\zeta]}{\varrho_3 \exp[(-8d)\zeta] + \cdots + \Gamma_3 \exp[(8q)\zeta]'} \tag{3.3}$$

$$\vdots$$

$$\vdots$$

$$\Psi^{(r)}(\zeta) = \frac{\tau_r \exp[-(c+(2^r-1)d)\zeta] + \cdots + \sigma_r \exp[(p+(2^r-1)q)\zeta}{\varrho_r \exp[(-2^r d)\zeta] + \cdots + \Gamma_r \exp[(2^r q)\zeta]'} \tag{3.4}$$

4. Explicit optical solutions of Eq. (1.1) using the Exp-function scheme

In this section, we will apply the Exp-function scheme to construct new explicit optical solutions of the nonlinear Chen-Lee-Liu equation (1.1). To this aim, we use the wave transform in [1]:

$$\Psi(z,t) = \Psi(\zeta) \exp[i(-kz + \omega t + \theta_0)],$$

to convert Eq. (1.1) to the following nonlinear ODE:

$$\rho^2 a \Psi'' + k(\vartheta^2 \beta + \alpha - \theta)\Psi^3 - (k^2 a + \lambda k + \omega)\Psi = 0, \tag{4.1}$$

whereρ, k, α, β and ϑ are constants and $\zeta = \rho(z - \vartheta t)$, for more details, see [1].

we suppose that the solution of Eq. (4.1) can be expressed as:

$$\Psi(\zeta) = \frac{w_{-c} \exp(-c\zeta) + \cdots + w_p \exp(p\zeta)}{s_{-d} \exp(-d\zeta) + \cdots + s_q \exp(q\zeta)}, \tag{4.2}$$

where c, d, p and q are positive integers.

To determine the values of c and d, we balance the linear term of lowest order Ψ'' in Eq. (4.1) with the lowest order nonlinear term Ψ^3. Simplifying the calculations, we have

$$\Psi'' = \frac{\tau_1 \exp[-(c+3d)\zeta] + \cdots}{\tau_2 \exp[(-4d)\zeta] + \cdots \quad -} \tag{4.3}$$

and

$$\Psi^3 = \frac{\tau_3 \exp[-(3c+d)\zeta] + \cdots}{\tau_4 \exp[(-4\,d)\zeta] + \cdots}, \tag{4.4}$$

where $\tau_i, (i = 1, \dots \dots, 4)$ are determined coefficients only for simplicity. From Eqs. (4.3) and (4.4), we obtain

$$-(c+3d) = -(3c+d) \Longrightarrow c = d. \tag{4.5}$$

Similarly, to determine values p and q, we balance the linear term of the highest order Ψ'' in Eq. (4.1) with the highest order nonlinear term Ψ^3. Simplifying the calculations, we have

$$\Psi'' = \frac{\dots + \sigma_1 \exp[(p+3q)\zeta]}{\dots + \sigma_2 \exp[(4q)\zeta]} \tag{4.6}$$

and

$$\Psi^3 = \frac{\dots + \sigma_3 \exp[(3p+q)\zeta]}{\dots + \sigma_4 \exp[(4q)\zeta]}, \tag{4.7}$$

where $\sigma_n, (n = 1, \dots \dots, 4)$ are determined coefficients only for simplicity. From Eqs. (4.6) and (4.7), we obtain

$$p + 3q = 3p + q \Rightarrow p = q. \tag{4.8}$$

Fow simplicity, we take the following cases:

Case 1.

If $c = d = 1$ and $p = q = 1$, so Eq. (4.2) reduces to

$$\Psi(\zeta) = \frac{w_{-1} \exp(-\zeta) + w_0 + w_1 \exp(\zeta)}{s_{-1} \exp(-\zeta) + s_0 + s_1 \exp(\zeta)}. \tag{4.9}$$

There are some free parameters in Eq. (4.9), for simplicity, we set $s_1 = 1$, so Eq. (4.9) reduces to

$$\Psi(\zeta) = \frac{w_{-1} \exp(-\zeta) + w_0 + w_1 \exp(\zeta)}{s_{-1} \exp(-\zeta) + s_0 + \exp(\zeta)}. \tag{4.10}$$

Substituting Eq. (4.10) into Eq. (4.1), and with the help of Maple software, we have

$$\frac{1}{\mathrm{E}}[\Omega_3 \exp(3\zeta) + \Omega_2 \exp(2\zeta) + \Omega_1 \exp(\zeta) + \Omega_0 + \Omega_{-1} \exp(-\zeta) + \Omega_{-2} \exp(-2\zeta) + \Omega_{-3} \exp(-3\zeta)] = 0. \tag{4.11}$$

where

$\mathrm{E} = (s_{-1} \exp(-\zeta) + s_0 + \exp(\zeta))^3$

$\Omega_3 = -\beta k \vartheta^2 w_1^3 - \alpha k w_1^3 + k\theta w_1^3 + ak^2 w_1 + k\lambda w_1 + \omega w_1,$

$\Omega_2 = -3\beta k \vartheta^2 w_0 w_1^2 + 2ak^2 w_1 s_0 + a\rho^2 w_1 s_0 - 3\alpha k w_1^2 + 3k w_0 \theta w_1^2 + ak^2 w_0 - a\rho^2 w_0 + 2k\lambda w_1 s_0 + k\lambda w_0$
$\quad + 2\omega w_1 s_0 + \omega w_0,$

$\Omega_1 = -3\beta k \vartheta^2 w_{-1} w_1^2 - 3\beta k \vartheta^2 w_0^2 w_1 + ak^2 w_1 s_0^2 - a\rho^2 w_1 s_0^2 + 2ak^2 w_0 s_0 + 2ak^2 w_1 s_{-1} + 2\rho^2 w_0 s_0$
$\quad + 4a\rho^2 w_1 s_{-1} - 3\alpha k w_{-1} r_1^2 - 3\alpha k w_0^2 w_1 + k\lambda w_1 s_0^2 + 3k\theta w_{-1} s_1^2 + 3w_0^2 k\theta w_1 + ak^2 w_{-1}$
$\quad - 4a\rho^2 w_{-1} + 2k\lambda w_0 s_0 + 2k\lambda w_1 s_{-1} + \omega w_1 s_0^2 + \lambda k w_{-1} + 2\omega w_0 s_0 + 2\omega w_1 s_{-1} + \omega s_{-1},$

$\Omega_0 = ak^2 w_0 s_0^2 + \lambda k w_0 s_0^2 - \beta k \vartheta^2 w_0^3 + 2ak^2 w_{-1} s_0 + 2ak^2 w_0 s_{-1} + 2k\lambda w_{-1} s_0 + 2k\lambda w_0 s_{-1} + 2\omega w_1 s_{-1} s_0$
$\quad - 3a\rho^2 w_{-1} s_0 + 6a\rho^2 w_0 s_{-1} + 2\omega w_{-1} s_0 + 2\omega w_0 s_{-1} + \omega w_0 s_0^2 - \alpha k w_0^3 + k\theta w_0^3$
$\quad + 2w_{-1} w_0 ak^2 w_1 + 2w_1 k w_{-1} \lambda w_0 - 3a\rho^2 w_1 s_{-1} s_0 - 6\alpha k w_{-1} w_0 w_1 + 6k\theta w_{-1} w_0 w_1$
$\quad - 6w_{-1} \beta k w_0 \vartheta^2 w_1,$

$$\Omega_{-1} = -3\beta k\vartheta^2 w_{-1}^2 w_1 - 3\beta k\vartheta^2 w_{-1} w_0^2 + ak^2 w_{-1} s_0^2 + 2ak^2 w_0 s_{-1} s_0 + ak^2 w_1 s_{-1}^2 - a\rho^2 w_{-1} s_0^2 + a\rho^2 w_0 s_{-1} s_0 - 4a\rho^2 w_1 s_{-1}^2 + 2ak^2 w_{-1} s + 4a\rho^2 w_{-1} s - 3\alpha k w_{-1}^2 w_1 - 3\alpha k w_0^2 + k\lambda w_{-1} s_0^2 + 2k\lambda w_0 s_{-1} s_0 + k\lambda w_1 s_{-1}^2 + 3k\theta w_{-1}^2 w_1 + 3k\theta w_{-1} + 2w_0^2 k\lambda w_{-1} s_{-1} + \omega w_{-1} s_0^2 + 2\omega w_0 s_{-1} s_0 + \omega w_1 s_{-1}^2 + 2\omega w_{-1} s_{-1},$$

$$\Omega_{-2} = -3\beta k\vartheta^2 w_{-1}^2 w_0 + 2ak^2 w_{-1} s_{-1} s_0 + ak^2 w_0 s_{-1}^2 + a\rho^2 w_{-1} s_{-1} s_0 - a\rho^2 w_0 s_{-1}^2 - 3\alpha k w_{-1}^2 w_0 + 2k\lambda w_{-1} s_{-1} s_0 + k\lambda w_0 s_{-1}^2 + 3k\theta w_{-1}^2 w_0 + 2\omega w_{-1} s_{-1} s_0 + \omega w_0 s_{-1}^2,$$

$$\Omega_{-3} = -\beta k\vartheta^2 w_{-1}^3 + ak^2 w_{-1} s_{-1}^2 - \alpha k w_{-1}^3 + k\lambda w_{-1} s_{-1}^2 + k\theta w_{-1}^3 + \omega w_{-1} s_{-1}^2.$$

Equating the coefficients of $\exp(i\zeta)$; $(i = 0, \pm1, \pm2, \pm3)$ to be zero, we have

$$\begin{bmatrix} \Omega_3 = 0, & \Omega_{-3} = 0, \\ \Omega_2 = 0, & \Omega_{-2} = 0, \\ \Omega_1 = 0, & \Omega_{-1} = 0, \\ \multicolumn{2}{c}{\Omega_0 = 0.} \end{bmatrix}, \tag{4.12}$$

On solving the above system of algebraic equations using Maple, we get the following results:

$$w_{-1} = \frac{(ak^2 + k\lambda + \omega)s_{-1}}{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}\ \ k(\beta\vartheta^2 + \alpha - \theta)},$$

$$w_0 = w_0, \qquad s_{-1} = s_{-1}, \qquad w_1 = \sqrt{\frac{ak^2 + k\lambda + \omega}{\beta k\vartheta^2 + \alpha k - k\theta}},$$

$$s_0 = \frac{w_0}{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}}. \tag{4.13}$$

where w_0 and s_{-1} are free parameters. Let $w_0 = -1,\ s_{-1} = 1.$ Therefore, Substitution these results into Eq. (4.10), we obtain the following explicit optical solution:

$$\Psi(z,t) = \left[\frac{\frac{(ak^2+k\lambda+\omega)\mathrm{e}^{-\rho(z-\vartheta t)}}{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}\ \ k(\beta\vartheta^2+\alpha-\theta)} - 1}{\mathrm{e}^{-\rho(z-\vartheta t)} - \frac{1}{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}} + \mathrm{e}^{\rho(z-\vartheta t)}} + \frac{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}\ \mathrm{e}^{\rho(z-\vartheta t)}}{\mathrm{e}^{-\rho(z-\vartheta t)} - \frac{1}{\sqrt{\frac{ak^2+k\lambda+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}} + \mathrm{e}^{\rho(z-\vartheta t)}} \right] \times \exp[i(-kz + \omega t + \theta_0)] \tag{4.14}$$

which is the explicit optical soliton solution of the nonlinear Chen-Lee-Liu equation when $c = d = 1$ and $p = q = 1$,.

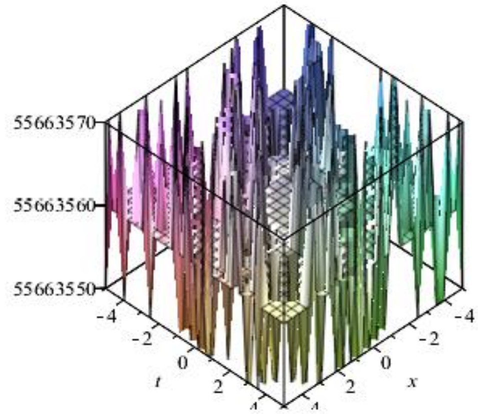

Figure 1. The profile of the optical-soliton solution (4.14)

with $\boldsymbol{\rho = 1,\ a = 2,\ k = -1,\ \omega = 3,\ \lambda = -1,\ \delta = 1,\ \theta = 1,\ \alpha = \frac{1}{2},\ \beta = \frac{1}{4},\ \vartheta = -1}$.

Case 2.

If $c = d = 1$ and $p = q = 2$, so Eq. (4.2) reduces to

$$\Psi(\zeta) = \frac{w_{-1}\exp(-\zeta)+w_0+w_1\exp(\zeta)+w_2\exp(2\zeta)}{s_{-1}\exp(-\zeta)+s_0+s_1\exp(\zeta)+s_2\exp(2\zeta)} \qquad (4.15)$$

There are some free parameters in Eq. (4.15), for simplicity, we set $s_2 = 1$ and $s_1 = 0$, therefore Eq. (4.15) becomes as follows:

$$\Psi(\zeta) = \frac{w_{-1}\exp(-\zeta)+w_0+w_1\exp(\zeta)+w_2\exp(2\zeta)}{s_{-1}\exp(-\zeta)+s_0+\exp(2\zeta)} \qquad (4.16)$$

Substituting Eq. (4.16) into Eq. (4.2), and with the help of Maple software, we have

$$\frac{1}{\mathrm{E}}[\Omega_6\exp(6\zeta)+\Omega_5\exp(5\zeta)+\Omega_4\exp(4\zeta)+\Omega_3\exp(3\zeta)+\Omega_2\exp(2\zeta)+\Omega_1\exp(\zeta)+\Omega_0+\Omega_{-1}\exp(-\zeta)$$
$$+\Omega_{-2}\exp(-2\zeta)+\Omega_{-3}\exp(-3\zeta)] = 0. \qquad (4.17)$$

where

$\mathrm{E} = (s_{-1}\exp(-\zeta)+s_0+\exp(2\zeta))^3$

$\Omega_6 = -\beta k\vartheta^2 w_2^3 - \alpha k w_2^3 + k\vartheta w_2^3 + ak^2 w_2 + k\lambda w_2 + \omega w_2,$

$\Omega_5 = -3\beta k\vartheta^2 w_1 w_2^2 - 3\alpha k w_1 w_2^2 + 3k\vartheta w_1 w_2^2 + ak^2 w_1 - a\rho^2 w_1 + k\lambda w_1 + \omega w_1,$

$\Omega_4 = -3\beta k\vartheta^2 w_0 w_2^2 - 3\beta k\vartheta^2 w_1^2 w_2 + 2ak^2 w_2 s_0 + 4a\rho^2 w_2 s_0 - 3w_2^2\alpha k w_0 - 3\alpha k w_1^2 + 3k w_2\theta w_0 w_2^2 + 3k\theta w_1^2 w_2$
$\quad + ak^2 w_0 - 4a\rho^2 w_0 + 2k\lambda w_2 s_0 + k\lambda w_0 + 2\omega w_2 s_0 + \omega w_0,$

$\Omega_3 = 6\rho^2 a w_1 s_0 - \beta k\vartheta^2 w_1^3 + 2ak^2 w_1 s_0 + 2k\lambda w_1 s_0 + 9\rho^2 a w_2 s_{-1} - 3k\alpha w_{-1} s_2^2 + 3k\theta w_{-1} w_2^2 + 2ak^2 w_2 s_{-1}$
$\quad + 2k\lambda w_2 s_{-1} - 3k\beta\vartheta^2 w_{-1} w_2^2 - 6w_0 k w_1 \alpha w_2 + 6k\theta w_0 w_1 w_2 - 6w_1 k w_0 \beta\vartheta^2 w_2 + \omega w_{-1}$
$\quad + ak^2 w_{-1} + k\lambda w_{-1} + k\theta w_1^3 - \alpha k w_1^3 + 2\omega w_1 s_0 + 2\omega w_2 s_{-1} - 9\rho^2 a w_{-1},$

$\Omega_2 = 2ak^2 w_0 s_0 + 2k\lambda w_0 s_0 - 4\rho^2 a w_2 s_0^2 + 4\rho^2 a w_0 s_0 - 3\alpha k w_0 w_1^2 + 3k\theta w_0 w_1^2 - 3k\alpha w_0^2 w_2 + 3k\theta w_0^2 w_2$
$\quad + 13a\rho^2 w_1 s_{-1} + 2ak^2 w_1 s_{-1} + 2k\lambda w_1 s_{-1} + k\lambda w_2 s_0^2 + ak^2 w_2 s_0^2 - 6w_{-1} k w_1 \alpha w_2$
$\quad + 6k\theta w_{-1} w_1 w_2 - 3\beta k\vartheta^2 w_0 w_1^2 - 3\beta k\vartheta^2 w_0^2 w_2 - 6w_{-1}\beta k w_1\vartheta^2 w_2 + \omega w_2 s_0^2 + 2\omega w_0 s_0$
$\quad + 2\omega w_1 s_{-1},$

$$\Omega_1 = -6k\beta\vartheta^2 w_{-1}w_0w_2 - 3k\beta\vartheta^2 w_{-1}w_1^2 + 11\rho^2 a w_2 s_{-1}s_0 - 6k\alpha w_{-1}w_0w_2 + 6kw_{-1}w_0\theta w_2 + 2ak^2 w_2 s_{-1}s_0$$
$$+ 2ak^2 w_0 s_{-1} + 2k\lambda w_{-1}s_0 + 2k\lambda w_0 s_{-1} + 2\omega w_2 s_{-1}s_0 - 3\alpha k w_0^2 w_1 + 3k\theta w_0^2 w_1 - a\rho^2 w_1 s_0^2$$
$$+ ak^2 w_1 s_0^2 + 2\lambda k w_2 s_{-1}s_0 - 3\beta k\vartheta^2 w_0^2 w_1 + \omega w_1 s_0^2 + 2\omega w_0 s_{-1} + 2\omega w_{-1}s_0 + k\lambda w_1 s_0^2$$
$$- 3\alpha k w_{-1}w_1^2 + 3k\theta w_{-1}w_1^2 - 2\rho^2 a w_{-1}s_0 + 13\rho^2 a w_0 s_{-1} + 2ak^2 w_{-1}s_0,$$
$$\Omega_0 = ak^2 w_0 s_0^2 + k\lambda w_0 s_0^2 - \beta k\vartheta^2 w_0^3 + 2\omega w_{-1}s_{-1} - 6\beta k\vartheta^2 w_{-1}w_0w_1 - 6\alpha k w_{-1}w_0w_1 + 6k\theta w_{-1}w_0w_1$$
$$+ 2ak^2 w_1 s_{-1}s_0 + 9a\rho^2 w_{-1}s_{-1} - 3k\alpha w_{-1}^2 w_2 + 3kw_{-1}^2\theta w_2 + \omega w_0 s_0^2 - \alpha k w_0^3 + k\theta w_0^3$$
$$+ 2k\lambda w_1 s_{-1}s_0 - 3a\rho^2 w_1 s_{-1}s_0 - 3k\beta\vartheta^2 w_{-1}^2 w_2 + ak^2 w_2 s_{-1}^2 + \lambda k w_2 s_{-1}^2 + 2ak^2 w_{-1}s_{-1}$$
$$+ 2k\lambda w_{-1}s_{-1} + 2\omega w_1 s_{-1}s_0 + \omega w_2 s_{-1}^2 - 9a\rho^2 w_2 s_{-1}^2,$$
$$\Omega_{-1} = -3\beta k\vartheta^2 w_{-1}^2 w_1 - 3\beta k\vartheta^2 w_{-1}w_0^2 + ak^2 w_{-1}s_0^2 + 2ak^2 w_0 s_{-1}s_0 + ak^2 w_1 s_{-1}^2 - a\rho^2 w_{-1}s_0^2 + a\rho^2 w_0 s_{-1}s_0$$
$$- 4a\rho^2 w_1 s_{-1}^2 - 3\alpha k w_{-1}^2 w_1 - 3\alpha k w_{-1}w_0^2 + k\lambda w_{-1}w_0^2 + 2w_0 w_{-1}k\lambda w_0 + k\lambda w_1 s_{-1}^2 + 3k\theta w_{-1}^2 w_1$$
$$+ 3k\theta w_{-1}w_0^2 + \omega w_{-1}s_0^2 + 2\omega w_0 s_{-1}s_0 + \omega w_1 s_{-1}^2,$$
$$\Omega_{-2} = -3\beta k\vartheta^2 w_{-1}^2 w_0 + 2ak^2 w_{-1}s_{-1}s_0 + ak^2 w_0 s_{-1}^2 + a\rho^2 w_{-1}s_{-1}s_0 - a\rho^2 w_0 s_{-1}^2 - 3\alpha k w_{-1}^2 w_0 + 2k\lambda w_{-1}s_{-1}s_0$$
$$+ k\lambda w_0 s_{-1}^2 + 3k\theta w_{-1}^2 s_0 + 2\omega w_{-1}s_{-1}s_0 + \omega w_0 s_{-1}^2,$$
$$\Omega_{-3} = -\beta k\vartheta^2 w_{-1}^3 + ak^2 w_{-1}s_{-1}^2 - \alpha k w_{-1}^3 + k\lambda w_{-1}s_{-1}^2 + k\theta w_{-1}^3 + \omega w_{-1}s_{-1}^2,$$

Equating the coefficients of $\exp(i\zeta)$; $(i = 0, \pm 1, \pm 2, \pm 3, 4, 5, 6)$ to be zero, we have

$$\begin{bmatrix} \Omega_3 = 0, & \Omega_{-3} = 0, & \Omega_6 = 0, \\ \Omega_2 = 0, & \Omega_{-2} = 0, & \Omega_5 = 0, \\ \Omega_1 = 0, & \Omega_{-1} = 0, & \Omega_4 = 0, \\ & \Omega_0 = 0, & \end{bmatrix}, \qquad (4.18)$$

On solving the above system of algebraic equations using Maple, we get the following results:

$$w_{-1} = \sqrt{\frac{ak^2 + \lambda k + \omega}{\beta k\vartheta^2 + \alpha k - k\theta}}\ s_{-1},$$
$$w_1 = 0, \qquad s_{-1} = s_{-1}, \qquad s_0 = s_0,$$
$$w_0 = \sqrt{\frac{ak^2 + \lambda k + \omega}{\beta k\vartheta^2 + \alpha k - k\theta}}\ s_0,$$
$$w_2 = \sqrt{\frac{ak^2+\lambda k+\omega}{\beta k\vartheta^2+\alpha k-k\theta}} \qquad (4.19)$$

where s_{-1} and s_0 are free parameters. Let $s_{-1} = 1, s_0 = -1$. Therefore, Substitution these results into Eq. (4.16), we obtain the following explicit optical solution

$$\Psi(z,t) = \left[\frac{\sqrt{\frac{ak^2+\lambda k+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}\ \mathrm{e}^{-\rho(z-\vartheta t)}}{\mathrm{e}^{-\rho(z-\vartheta t)} - 1 + \mathrm{e}^{2\rho(z-\vartheta t)}} + \frac{\sqrt{\frac{ak^2+\lambda k+\omega}{\beta k\vartheta^2+\alpha k-k\theta}} + \sqrt{\frac{ak^2+\lambda k+\omega}{\beta k\vartheta^2+\alpha k-k\theta}}\mathrm{e}^{2\rho(z-\vartheta t)}}{\mathrm{e}^{-\rho(z-\vartheta t)} - 1 + \mathrm{e}^{2\rho(z-\vartheta t)}}\right]$$
$$\times \exp[i(-kz + \omega t + \theta_0)] \qquad (4.20)$$

which is the explicit optical soliton solution of the nonlinear Chen-Lee-Liu equation when $c = d = 1$ and $p = q = 2$,.

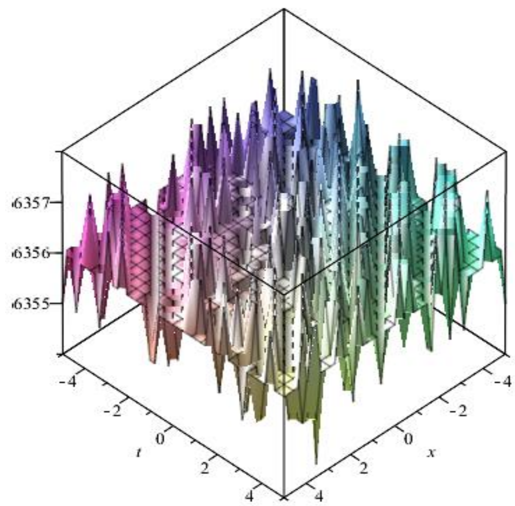

Figure 2. The profile of the optical-soliton solution (4.20) with $\rho = 1,\ a = 3,\ k = -1,\ \omega = 2,\ \lambda = -1,\ \delta = 1,\ \theta = 1,$ $\alpha = \frac{1}{4},\ \beta = \frac{1}{2},\ \vartheta = -1$.

5. Conclusions

The Exp-function scheme used in this article was successful and effective to obtain new explicit optical solutions to the nonlinear Chen-Lee-Liu equation. Many authors have used this analytical scheme and described it as straightforward, concise and powerful tool for solving nonlinear wave equations in mathematical physics.

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