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استخدام التحكم الأمثل بدون خسارة المال في المستقبل

إبتسام علي إبراهيم – أحلام سعد عبدالله
قسم الرياضيات، كلية العلوم، جامعة طبرق، ليبيا
ebtessamaali4@gmail.com

Using optimal control without loss the money in future

Ebtessam Ali Ebrahim – Ahlam.S.Abdulla
Department of Mathematics, Faculty of Science, University of Tobruk, Libya

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الملخص:

الغرض من كتابة هذا الموضوع هو لاستعراض دراسة مبسطة وقصيرة لمعرفة كيفية تضخيم اموال شخص ما باستخدام التحكم الامثل من دون خسارة والغرض من التحكم هنا هو تضخيم المردود المتوقع في المستقبل يمكن ملاحظة في هذه الدراسة نتيجة قيمة ومثالية للغاية أذا وجدنا القرار الامثل الذي سيجلب بعض المدخلات الايجابية في المستقبل نحن نعلم ان تضخيم الاموال دون خسارة هو في الواقع جزء هام في حياتنا لنفترض أن حالة النظام في الوقت المحدد يتم وصفها بواسطة عملية الايتو والتي لها الصيغة

⟦dX⟧ _t= ⟦dX⟧ _t^u=b(t,X_t,u_t)dt+σ (t,X_t,u_t) ⟦dB⟧ _t (1)

يفترض التحكم في ماركوف انه لا يعتمد علي نقطة البداية فالقيمة التي نختارها في ذلك الوقت تعتمد فقط علي النظام في هذا الوقت .

الكلمات الدالة: ماركروف كنترول،عملية ايتو ، براون موشن،جاكوبي بليمان،التحكم الأمثل.

Abstract

The purpose of writing this paper is to demonstrate a short study to learn how the person to maximize his money by using an optimal control without loss. The purpose of the control is to maximize the expected payoff in the future . We have in this paper very valuable result .

If we have found an optimal decision that will bring some positive inpute in future . We know that to maximize our money with out losing is actually very necessary part in our life.

Suppose that the state of a system at time t is described by ito process X_t of the form

$$dX_t = dX_t^u = b(t, X_t, u_t)dt + \sigma(t, X_t, u_t)dB_t \qquad (1)$$

Markov control is assume that u does not depend on the starting point $y = (s, x)$, the value we choose at time t only dependes on of the system at this time.

Keywords: Markov control – Ito process–Brownain motion–Hamilton – Jakobi bellman– optimal control

Introduction

It will be identify a function $u;\ R^{n+1} \to U$.

where

$X_t \in R^n, b: R \times R^n \times U \to R^n, \sigma: R \times R^n \times U \to R^n$ and B_t is $m-$ dimensional Brownian motion. Where $u_t = u(t, w)$ is a stochastic process.

because our decision at time t should be based on what has happened up to time t , the function $w \to u(t, w)$ must be measurable with respect to the filtration F_t^m, $i.e$ the process u_t must be F_t^m adapted.

Suppose that the process X_t satisfying the equation (1),

let $X_h^{s,x}$ be the solution for the equation (1) such that

$$X_h^{s,x} = x + \int_s^h b(r, X_r^{s,x}, u_r)\, dr + \int_s^h \sigma(r, X_r^{s,x}, _{U_r})dB_r; \quad h \geq s$$

$Q^{s,x}$ is the probability law of $Y_t = \left(s+t, X_{s+t}^{s,t}\right)t \geq 0$, P^0 is the probability law of B_t starting at 0, such that

$$Q^{s,x}[X_{t1} \in F_1, \dots, X_{tk} \in F_k] = P^0[X_{t1}^{s,x} \in F_1, \dots, X_{tk}^{s,x} \in F_k] \qquad (2)$$

for $s \leq t_i, F_i \subset R^n; 1 \leq i \leq k, k = 1,2 \dots$

Let $f : \mathbb{R} \times \mathbb{R}^n \times U \to \mathbb{R}$ the utility rate or profit rate function and

$g; \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ (the bequest function) be given continuous function ,

let G(the solvency set) and let T (the bankruptcy time) be the first time from G for the process $\{X_r^{s,x}\}$ mean that

$$T = T^{s,x}(w) = inf\left\{r > s; \left(r, X_r^{s,x}(w)\right) \notin G\right\} \leq \infty.$$

Assume

$$E^{s,x}\left[\int_s^T |f^{ur}(r, X_r)|dr + |g(T, X_T)|_{X_{\{T<\infty\}}}\right] < \infty. \qquad (3)$$

We define the performance function $J^u(s, x)$ by

$$J^u(s, x) = E^{s,x}\left[\int_s^T f^{ur}(r, X_r)dr + g(T, X_T)_{X_{\{T<\infty\}}}\right]. \qquad (4)$$

If we introduce the notation $Y_t = (s+t, X_{s+t}^{s,x})$ to make it easier and substitution this in equation (1) to obtain

$$dY_t = dY_t^u = b(Y_t, u_t)dt + \sigma(Y_t, u_t)dB_t. \qquad (5)$$

Where

$$\tau_G := \inf\{t > 0;\ Y_t \notin G\} = T - s \quad . \qquad (6)$$

Define

τ_G is the first exist time from the set G of a process Y_t.

Moreover

$$g(T, X_T) = g(Y_{T-s}) = g(Y_{\tau G}).$$

Therefore the performance function can be written as follows,

whith $y = (s, x)$

$$J^u(y) = E^y\left[\int_0^{\tau G} f^{ut}(Y_t)dt + g(Y_{\tau G})x_{\tau G < \infty}\right] \qquad (7).$$

The problem is to find the number $\Phi(y)$ and control

$$u^* = u^*(t, w) \in \mathcal{A}$$

such that

$$\Phi(y) := sup_{\,u(t,w)} J^u(y) = J^{u^*}(y).$$

The supremum is taken over a given family $\mathcal{A}$ of a admissible controls, u^* is called an optimal control if it is exists and Φ is called the optimal performance or the value function.

The Hamilton– Jacobi–Bellman Equation

The Hamilton – Jacobi – Bellman Equation is central result in optimal theory, we want to consider Markov controls only $u = \big(t, Xt(w)\big)$ define

$$a_{ij} = \frac{1}{2}\sigma(x)\sigma^T(x).$$

Introducing $Y_t = (s+t, X_{s+t})$ and the equation becomes

$$dY_t = b\big(Y_t, u(Y_t)\big)dt + \sigma(Y_t, u(Y_t))dB_t\,. \qquad (8)$$

For $v \in U$ and $\phi \in C_0^2\,(\mathbb{R} \times \mathbb{R}^n)$ define

$$(L^v\phi)(y) = \frac{\partial\phi}{\partial s}(y) + \sum_{i=1}^{n} b_i(y, v)\frac{\partial\phi}{\partial x_i} + \sum_{i,j=1}^{n} a_{ij}(y, v)\frac{\partial^2\phi}{\partial x_i \partial x_j}\,. \qquad (9)$$

The first fundamental and consequence result in stochastic control theory is the following:

Theorem: (The Hamilton– Jacobi – Bellman HJB (I))

$$Define$$

$$\Phi(y) = Sup\{J^u(y): u = u(Y)\}.\ Markov\ Control.$$

Assume that $\Phi \in C^2(G) \cap C(G^-)$ satisfies

$$E^y\left[|\phi(Y_\alpha)| + \int_0^\alpha |L^v(Y_t)|dt\right] < \infty$$

for all bounded stopping times $\alpha \le \tau_G$, for all $y \in G$ and all $v \in U$.

Furthermore an optimal Markov control u^* exists and ∂G is regular for

$Y_t^{u^*}\ in$ (7) .

Then

$$Sup\{f^v(y) + (L^v\Phi)(y)\} = 0 \ ,\forall\, y \in G \qquad (10)$$

and

$\Phi(y) = g(y)$,$\forall$ $y \in \partial G$. (11)

The HJB(I) equation state that if an optimal control u^* exists ,then we know that its value v at the point y is a point v where the function

$$v \to f^v(y) + (L^v\Phi)(y); v \in U$$

attains its maximum also states that it is necessary and sufficient that

$v = u^*(y)$ is the maximum of function.

Theorem (The HJB II):

Let ϕ be a function , for all $v \in U, such\ that$

$$f^v(y) + (L^v\phi)(y) \le 0; \quad y \in G \qquad (12)$$

with bounded values

$$\lim_{t\to tG} \phi(Y_t) = g(Y_{tG}).x\{tG < \infty\} \qquad (13)$$

Also, such that

$$\{\phi(Y_t); t\ \ stopping\ \ time\ ,\ \ t \le t_G\} \qquad (14)$$

is uniformly $Q^y -$ integrable for all Markov controls u and all $y \in G. Then$

$$\emptyset(y) \ge J^u(y)$$

for all Markov controls $u\ and\ all\ y \in G$.

likewise, if for each $y \in G$ we have found $u_0(y)$ such that

$f^{u_0(y)}(y)$ $+L^{u_0(y)}(y) = 0$ (15)

As well as

$$\{\phi(Y^{u_0}); t\ \ stopping\ time\ \ , t \le t_G\} \qquad (16)$$

is uniformly $Q^y -$ integrable for all $y \in G$ then $u_0 = u_0(y)$ is a Markov control such that $\phi(y) = J^{u_0}(y)$ and hence if u_0 is admissible then u_0 must be an

optimal control and $\phi(y) = \Phi(y)$.

The HJB equations (I),(II) provide a nice solution to the stochastic control problem in the case where only Markov controls are considered.

Theorem:

Let

$\Phi_M(y) = sup\{J^u(y); u\, u(Y)\;\; Markov\ control\}$

and

$\Phi_a(y) = sup\{J^u(y); u = u(t,w)\mathcal{F}_t^m - adapted\ control\}$

Suppose there exists an optimal Markov control $u_0 = u_0(y)$

for the Markov control problem $(i.e\ \Phi_M(y) = J^{u_0}(y) for\ all\ y\ \in G\ )$ such that

all the boundary points of G are regular with respect to $Y_t^{u_0}$ and that Φ_M is a boundary function in $C^2\ (G) \cap C(G)$ satisfying

$$E^y\left[|\Phi_M(Y_\alpha)| + \int_0^\alpha |L^u\Phi_M(Y_t)|\,dt\right] < \infty \qquad (17)$$

for all bounded stopping times $\alpha \le t_G$,all adapted controls u and all $y \in G$.

Then

$$\Phi_M(y) = \Phi_a(y)\ \ for\ all\ y\ \in G.$$

proof

Let ϕ be a bounded function in $C^2(G) \cap C(G)$ satisfying the equation (17)

and

$f^v(y) + (L^v\phi)(y) \le 0\ for\ all\ y\ \in G, v \in U \qquad (18)$

and

$$\phi(y) = g(y) \qquad for\ all\ \ y\ \in\ \partial G. \qquad (19)$$

Let $u_t(w) = u(t,w)$ be an $\mathcal{F}_t^m-$ adapted control . Then Y_t is an Ito process giveng by

$$dY_t = b(Y_t,u_t)dt + \sigma(Y_t,u_t)dB_t$$

so by the lemma of ito process with

$$T_R = min\{R, t_G; \inf t > 0; |Y_t| \ge R\}\ for\ all\ R < \infty.$$

$$E^y[\phi(Y_{TR})] = \phi(y) + E^y\left[\int_0^{T_R}\left(L^{u(t,w)}\ \phi\right)(Y_t)dt\right],$$

where

$$\left(L^{u(t,w)}\phi\right)(y) = \qquad \frac{\partial\phi}{\partial t}(y) + \sum_{i=1}^n b_i\big(y,u(t,w)\big)\frac{\partial\phi}{\partial x_i}(y) + \sum_{i,j=1}^n a_{ij}(y,u(t,w))\frac{\partial^2\phi}{\partial x_i\partial x_j}(y),$$

thus by the equation (17) and (18) this gives

$$E^y[\phi(Y_{TR})] \leq \phi(y) - E^y\left[\int_0^{T_R} f\big(Y_t, u(t,w)\big)\, dt\right].$$

Letting $R \to \infty$ we obtain

$$\phi(y) \geq J^u(y) \qquad (20)$$

the function $\phi(y) = \Phi_M(y)$ satisfies the equation (18) and (19).

So by the equation (19) we have $\Phi_M(y) \geq \Phi_a(y)$ and the theorem follows.

The Hamilton – Jacobi–Bellman equation (I) Says that :If there is an optimal control u^* then

$$f^v(y) + (L^v\Phi)(y) = 0,$$

however it cannot be said that if v satisfies

$$f^v(y) + (L^v\Phi)(y) = 0,$$

then v is an optimal control . The HJB (II) says if the equation

$$f^{u_0(y)}(y) + \big(L^{u_0(y)}\phi\big)(y) = 0$$

holds and hence u_0 is admissible then u_0 be an optimal control.

The theorem on Markov control explains that it does not matter restricting to Markov control because we must not have information about the history .We can obtain to a good an control without have information about the past , we always obtain as good performance with a Markov control as with an arbitrary $\mathcal{F}_t^{(m)}$ – adapted control .

Conclusion:

At the conclusion of this short paper, which highlighted an optimal control problems. The prominent purpose of this study to Maximize the expectation of payoff in future and we use optimal control for optimization of their activity .

Optimal control theory is easy to give more information to commiserated the mathematical reasons of the decision making process in our world.

writing down the HJB equations provide a nice solution to the Stochastic control.

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