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Properties Of Pure Neutron Matter and Three-Body Forces

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خواص المادة النيوترونية النقية وقوى ثلاثية الجسيمات

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الملخص:

نتائج حسابات المادة النيوترونية النقية الباردة والساخنة باستخدام تقريب بروكنر- هاتري- فوك مضاف لجهد سكيرم لجسيمين المعتمد على الكثافة الذي يكافئ قوى ثلاثية الجسيمات باستخدام جهد نووي حديث أرجونا ف 18. لقد قمنا بحساب الطاقة الحرة ومعادلة الحالة والضغط للمادة النيوترونية النقية عند درجات حرارة (0 ، 8 و 12) مليون إلكترون فولت وتم مقارنة هذه النتائج بالتقديرات النظرية السابقة، ولقد تحصلنا باستخدام هذه الطريقة على تطابق جيد مقارنة مع حسابات بالدو وفيريرا لسنة (1999) .

الكلمات الدالة: الطاقة الحرة، الضغط، المادة النووية النقية، تقريب بروكنرهاتري فوك، معادلة الحالة.

Abstract

Results of cold and hot pure neutron matter (PNM) calculations are presented. The Brueckner-Hartree-Fock (BHF) approximation plus two body density dependent Skyrme potential which is equivalent to three body interaction are used. Argonne V_{18} nucleon- nucleon (NN) potential is used in the framework of (BHFA). The Free energy (EOS) and the pressure of Pure Neutron Matter are computed at (T = 0, 8 MeV, 12 MeV), the results are compared with previous theoretical estimates.
Good agreement is obtained in comparison with M. Baldo and L. S. Ferreira (1999) calculation.

Keywords: PNM , BHFA , EOS, Free energy, Pressure.

Introduction

The aim of nuclear matter and neutron matter theory is to obtain empirically known properties, such as the free energy, symmetry energies, compressibility etc. The use of Brueckner theory for finite nuclei [1] is not suitable and that is why effective potentials are suggested to overcome the difficulties which arise from using the realistic potentials, in this connection the Skyrme interaction [2] proves to be good.

In a special representation, the Skyrme interaction contains a two-body short range and three-body zero range part , which is equivalent to a two-body density dependent interaction [3]. Mansour et al. [4] suggested a similar treatment for neutron matter by using a term which depends on the two densities of neutrons of spin up and down.

It is well known that the Skyrme interaction is a simple and useful potential for describing the properties of neutron matter.
On a microscopic basis the equation of state (EOS) of symmetric nuclear matter has been extensively studied within the variational approach[5-8] as well as relativistic [9-15] and non relativistic [16, 17] Brueckner–Hartree–Fock (BHF) theories. The predictions of non relativistic microscopic approaches (including both the BHF and variational approaches) based on pure two-body nucleon–nucleon (*NN*) forces (2BF) do not give the empirical saturation point of symmetric nuclear matter(Coester band) [18]. In order to improve the nuclear saturation.
two lines have been followed. One is the development of the relativistic mean field (RMF) theory [19] and Dirac–Brueckner–Hartree–Fock (DBHF) approach [10, 20-24]. The DBHF has been successful in describing the saturation properties of symmetric nuclear matter (SNM). However, still there are some problems remaining unsettled, such as the negative energy state problem, the ambiguities related to the decomposition of the effective reaction matrix into covariant amplitudes due to various approximations introduced for reducing the four-dimensional Bethe–Salpeter equation to the corresponding three-dimensional one. In the second line the medium effects are taken into account by phenomenological or microscopic three-body forces (3BF) within non-relativistic contexts. Calculations with phenomenological 3BF have been performed both in the framework of the variational approach [5-7] and the BHF approximation [25–28]. The basic input quantity in the BHF calculation is the *NN* interaction in free space.

Pure neutron matter (PNM) is the theoretical starting point for the (EOS) of neutron stars, the EOS connects free energy, energy density and pressure to determine the star's structure, Neutron stars are macroscopic realizations of the microscopic free energy and pressure of pure nuclear matter (PNM), low density PNM used in multifragmentation reactions and pion production ratio. In short the (PNM) is the theoretical foundation for interpreting neutron star observation (masses, radii, cooling), heavy ion collisions, structure and testing fundamental many-body quantum physics.

The goal of this work is calculating the free energy and pressure of pure nuclear matter theoretically.

We adopted the modern Argonne V_{18} potential [29]. the present work we add the corrections of the three-body forces using an equivalent density dependent two body forces of Skyrme type. Hot systems are also considered for small temperatures. In the next section we give a brief description of the method of calculation. Section 3 is devoted for a presentation of our main results.

2. Theory:

Here we start with a short review of the theoretical framework:
The microscopic Brueckner–Bethe–Goldstone description of nuclear matter is based on a linked cluster expansion of the energy per nucleon of nuclear matter [28].The basic ingredient is the Brueckner reaction matrix *G*, which is the solution of the Bethe–Goldstone equation :

$$G(\omega) = V + V \frac{Q}{\omega - H_\circ + i\eta} G(\omega). \quad (1)$$

Here, ω is the starting energy which is usually the sum of the single-particle energies of the states of the interacting nucleons,

$$\omega = e(k) + e(k'). \quad (2)$$

V is the bare *NN* potential, η is an infinitesimal small number, H_o is the unperturbed energy of the intermediate scattering states, *e* is the single-particle energy, and *Q* is the Pauli projection operator; it projects out states with two nucleons above the Fermi level, it is given by:

$$Q(k, k') = (1-\Theta_f(k))(1-\Theta_f(k')), \qquad (3)$$

where $\Theta_f(k) = 1$ for $k < k_f$ and zero otherwise, $\Theta_f(k)$ is the occupation probability of a free Fermi gas with Fermi momentum k_f..

In the Brueckner–Goldstone expansion, the average binding energy per nucleon is expanded in a series of terms as the following:

$$\frac{E(k)}{A} = \langle\hat{T}\rangle + \langle\hat{G}\rangle = \sum_{k}\frac{\hbar^2k^2}{2m} + \frac{1}{2}\sum_{k,k'<k_f}\langle kk'|\,G(e(k)+e(k'))\,|kk'\rangle, \quad (4)$$

where |kk' > refer to antisymmetrized two-body states. This first order is known as the Brueckner–Hartree–Fock (BHF) approximation. To completely determine the average binding energy one has to define the single-particle potential *U (k)* which contributes to the single-particle energies appearing in the *G*-matrix elements. The structure of the

expression (4) suggests choosing the following BHF single-particle potential:

$$U(k) = \sum_{k'<k_f}\langle kk'|\,G(e(k)+e(k'))\,|kk'\rangle \qquad (5)$$

$$\frac{E(k)}{A} = \sum_{k<k_f}\left\{\frac{\hbar^2k^2}{2m} + \frac{1}{2}U(k)\right\} = \frac{4}{\rho}\int_0^{k_f}\frac{4\pi k^2}{(2\pi)^3}\left(\frac{\hbar^2k^2}{2m} + \frac{1}{2}U(k)\right)dk =$$

$$\frac{3\hbar^2k_f^2}{10m} + \frac{3}{2k_f^3}\int_0^{k_f}k^2\,dk\,U(k), \qquad (6)$$

where ρ is the matter density.

The G-matrix itself depends on *U (k)* through the starting energy *ω*, defined in Eq. (2), and the lowest-order approximation (4) along with choice (5) for the single-particle potential is often known as the lowest-order Brueckner theory. The single particle energy *e (k)* is defined as:

$$e(k) = T + U(k) = \frac{\hbar^2k^2}{2m} + U(k), \qquad (7)$$

where T is the kinetic energy. The conventional choice for the single-particle potential has been to take the BHF potential (Eq. (5)) for hole states ($k < k_f$) and zero for particle states ($k > k_f$), thus introducing as:

$$U(k) = \begin{cases} \sum\limits_{k' \le k} \langle kk' | G(e(k)+e(k')) | kk' \rangle & k \le k_f \\ 0 & k > k_f \end{cases} \qquad (8)$$

Eqs. (1) and (7) represent the main equations that one has to solve self-consistently. In order to achieve saturation in nuclear matter one has to add three-body interaction terms or a density-dependent two-nucleon interaction. We have chosen it following the notation of the Skyrme interaction to be of the form:

$$v(\mathbf{r}_1,\mathbf{r}_2) = \sum_i t_i\,(1+y_i p_\sigma)\rho^{\alpha_i}\,\delta(\mathbf{r}_1-\mathbf{r}_2)$$

, (9)

where r_1 and r_2 are the position vectors for the particle 1 and particle 2 respectively, P_σ is the spin exchange operator, ρ is the matter density t_i ,y_i, and α_i are parameters.

For various values of α_i (typically α_i = 1/3, 2/3, 0.5, and 1) we have fitted t_i and y_i in such a way that a BHF calculation plus the contact terms yield the empirical saturation point for symmetric nuclear matter.

Having obtained the energy per particle *E/A* for zero temperature, the free energy $F = E/A - aT$. calculated at temperature *T* using the expression of the level density [30]. Among the different sets of parameters α_i proposed here the best results were
obtained for two terms of the above summation where α_1 and α_2 are equal to 1/3 and 2/3 respectively.

3. Results and Discussion:

3.1. Calculation of the EOS :

The EOS is the relationship between energy per nucleon and Fermi momentum k_F or density. The results are shown in Fig-1 at T= 0, 8, 12 MeV. where the energy per particle (F/A) in MeV ploted against density ρ in fm^{-3}

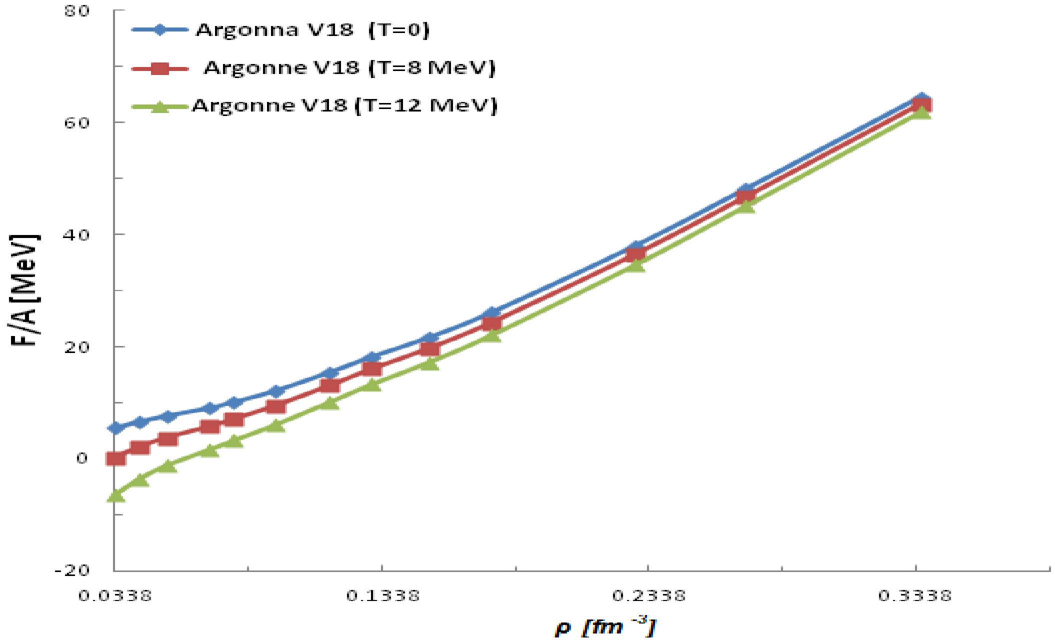

Fig. 1. F/A of pure neutron matter as a function of density at (T=0, 8 and 12 MeV) using Argonne V_{18} potential.

for pure neutron matter using Argonne V_{18} potential and the parameters of the contact potential are given in table (1).

Table (1): Interaction parameters of Argonne V_{18} potential

t_1	t_2	y_1	y_2
- 969.3	1521.1	0.2984	- 0.2076

A comparison is made with M. Baldo and L. S. Ferreira calculation [31, 32]
v_{14} +TNI realistic potential. The results are identical with M. Baldo and L. S. Ferreira at low densities.

3.2. Calculation of the free energy:

The free energy of the pure neutron matter is defined by:

$$F = E_{T=0} - a T^2 \qquad (10)$$

$$a = (\pi^2/2)\,(m^*/\hbar^2 k^2_F) \qquad (11)$$

$$k^2_F = (3\pi^2 \rho)^{2/3} \qquad (12)$$

where *F* is the free energy of the system,$E_{T=0}$ is the total energy at T = 0,
(a) is the level density of the system (30) and ħ is the Planck"s constant divided by 2π
where *m** is the effective mass of the nucleon. The results are shown in the Fig-2 at T= 0, Fig-3 at T= 8 MeV and Fig-4 at T= 12 MeV in comparison with M. Baldo and L. S. Ferreira calculation [31].

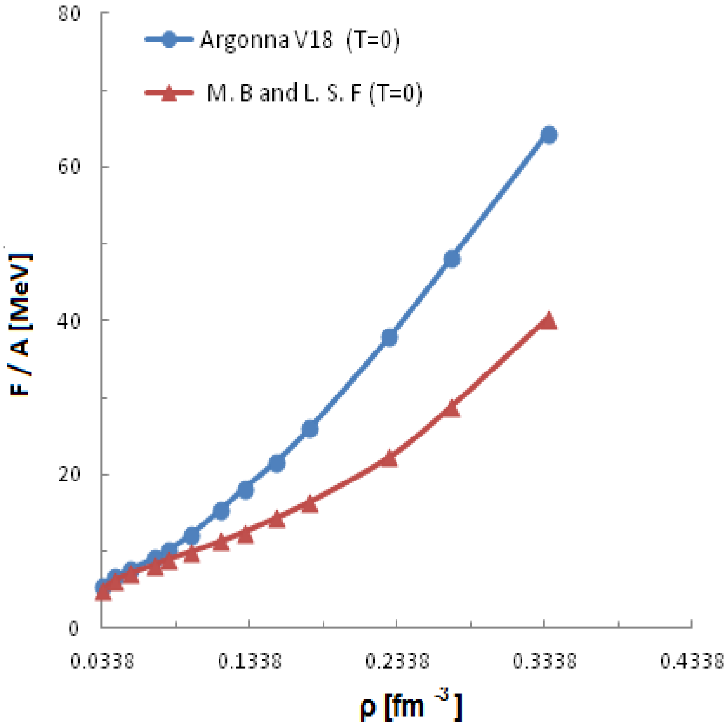

Fig. 2. F / A in MeV for pure neutron matter at (T= 0) as a function of density using Argonne V_{18} potential in comparison with M. B and L. S. F [31]

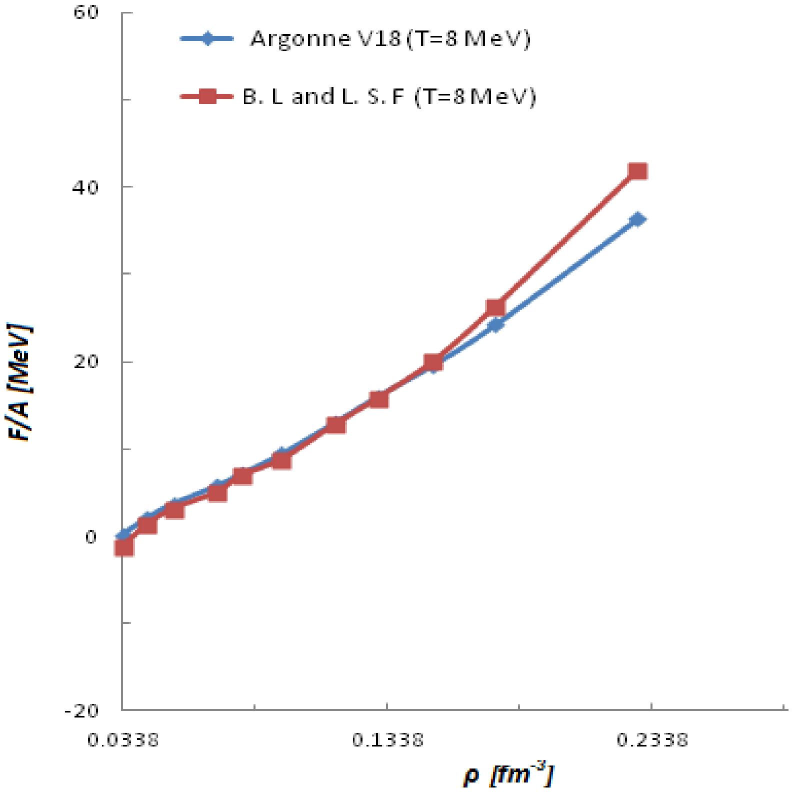

Fig. 3. F / A in MeV for pure neutron matter at (T=8 MeV) as a function of density using Argonne V_{18} potential in comparison with M. B and L. S. F [31]..

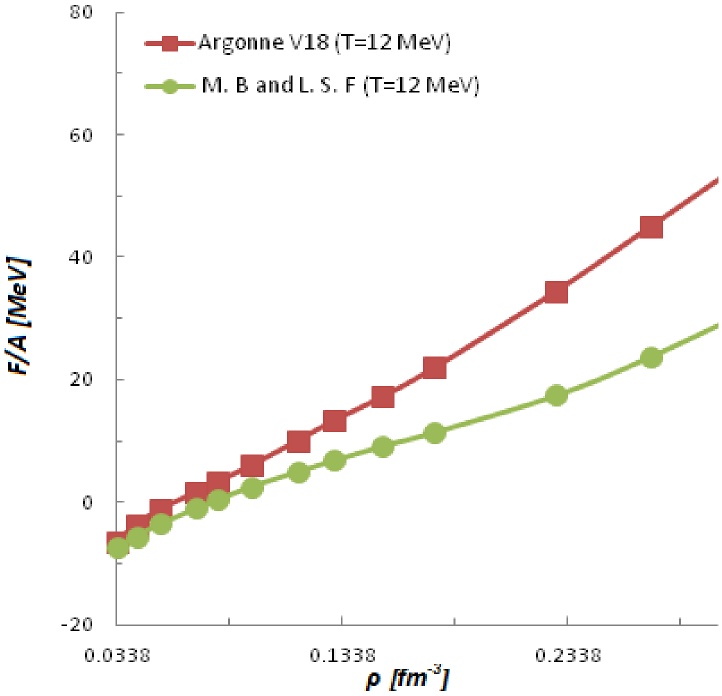

Fig. 4. F / A in MeV for pure neutron matter at (T=12 MeV) as a function of density using Argonne V_{18} potential in comparison with M. B and L. S. F [31].

3.3. Calculation of the pressure :

The pressure of pure neutron matter is defined as

$$P(\rho) = \rho^2 \, \partial \, (F/A) \, (\rho) \, / \, \partial \, \rho \qquad (13)$$

by differentiating equation (10) we get

$$P = P_{T=0} + (\pi^2 \, m^* \, k^2_F \, T^2 \, / \, 6 \, \hbar^2) \qquad (14)$$

The equation (14) is used to calculate the pressure of the pure neutron matter .

where P is the pressure of the system, $P_{T=0}$ is the total pressure at T=0, , ħ is the Planck's constant divided by 2π and m* is the effective mass of the neutron.

The results are shown in the Fig. 5 at T=0, Fig. 6 at T= 8 MeV and Fig. 7 at T =12 MeV in comparison with M. Baldo and L. S. Ferreira calculation [32].

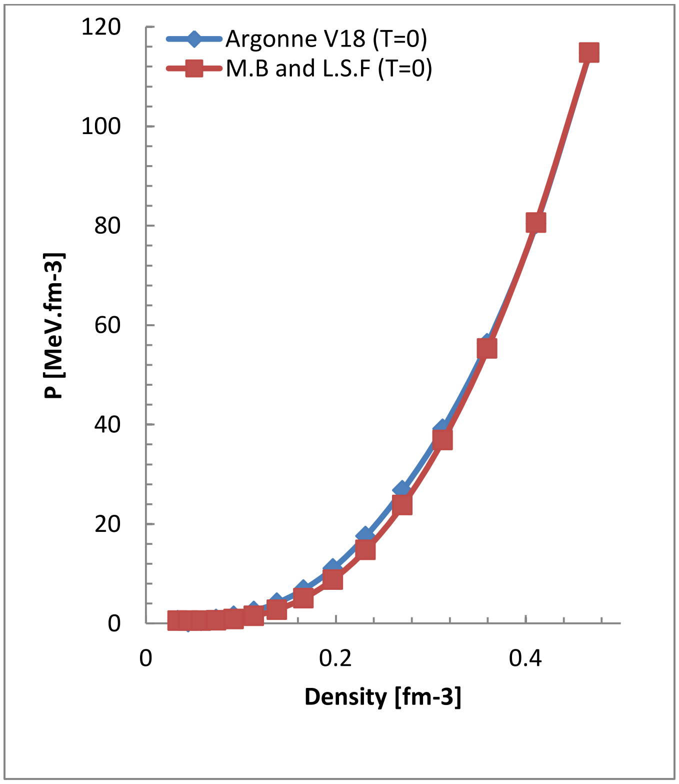

Fig.5: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T= 0) as a function of density in [fm^{-3}] using ArgonneV$_{18}$ potential in comparison with M. B and L. S. F [32].

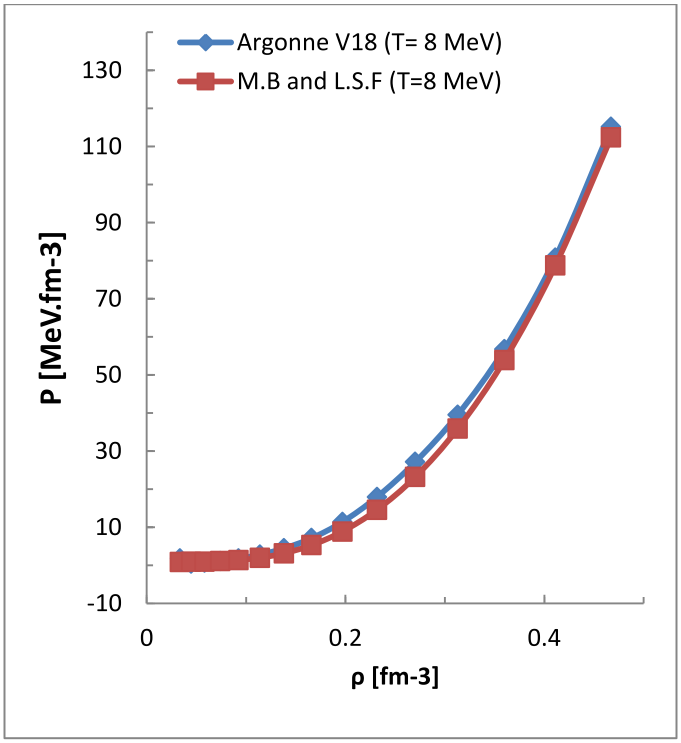

Fig.6: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T=8 MeV) as a function of density in [fm^{-3}] using Argonne potentialV_{18} in comparison with M. B and L. S. F [32].

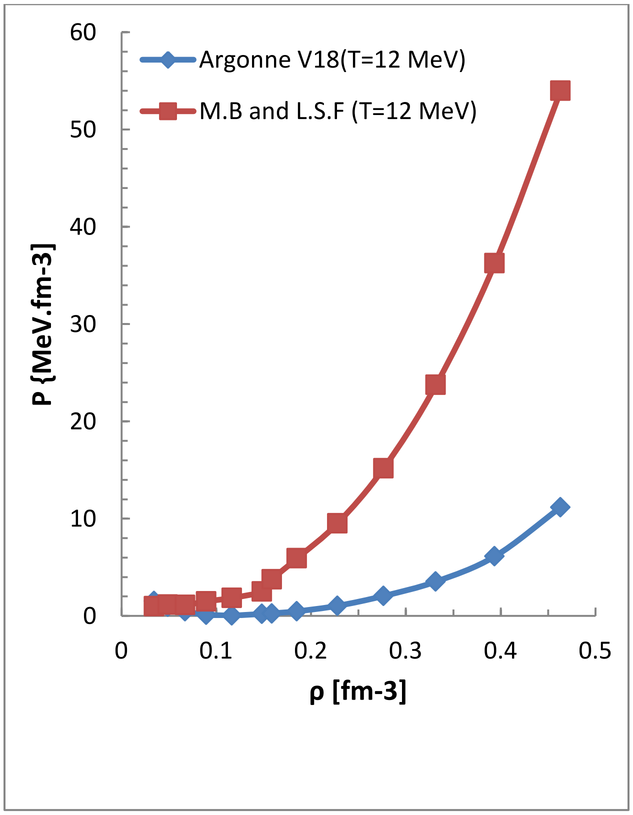

Fig.7: Pressure of the pure neutron matter in [MeV.fm^{-3}] at (T=12 MeV) as a function of density in [fm^{-3}] using Argonne potentialV_{18} in comparison with M. B and L. S. F [32].

4. SUMMARYAND CONCLUSION:

The bulk properties of pure neutron matter (PNM) are computed such as the equation of state EOS, free energy and pressure as a function of density.

The calculations of the above properties for neutron matter are made by using BHF interaction + two body density dependent Skyrme interaction which is equivalent to three body interaction. Modern *NN* interaction Argonne V_{18} potential is used in order to analyze the dependence of the results on the nuclear interaction.

The EOS and the pressure for pure neutron matter are calculated at(T = 0).

The free energy and the pressure of pure neutron matter at (T = 8 MeV and 12 MeV) have been calculated. The results are good in comparison with M. B and L. S. F calculation. One concludes that the calculations are only suitable at low density.

By this method good agreement is obtained in comparison with previous theoretical using v_{14}+TNI realistic potential calculation of M. B and L. S. F estimates.

References:

- K. A. Brueckner, J. L. Gammel and J. T. Kubis, Phys. Rev. 118, 1095 (1960).
- T. H. R. Skyrme, Phil. Mag. 1, 1043 (1956)
- J. Dabrowski, Nukleonika 21, 143, (1977).
- H. M. M. Mansour and J. W. Guirguis, Ann. der. Phys, 7, 260 (1989).
- R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C 38, 1010 (1988).
- A. Akmal and V. R. Pandharibande, Phys. Rev. C56, 2261(1997).
- A. Akmal and V. R. Pandharibande, Phys. Rev. C58, 1804(1998).
- G. H. Bordbar and M. Modarres, Phys. Rev. C 57, 714 (1998).
- L. Engvik, M. Hjorth-Jensen, E. Osnes, G. Bao, and E. Ostgaard, Phys. Rev. Lett. 73, 2650 (1994).
- B.T. Haar and R. Malfliet, Phys. Rev. Lett. 59, 1652 (1987); R.Malfliet, Nucl. Phys. A 488, 721c (1988).
- L. Engvik, M. Hjorth-Jensen, E. Osnes, G. Bao, and E.Ostgaard, Astrophys. J. 469, 794 (1996).
- H. Huber, F. Weber, and M.K. Wiegel, Phys. Rev. C 51, 1790 (1995); Phys. Rev. C 57, 3484 (1998).
- C. H. Lee, T.T. S. Kuo, G. Q. Li, and G. E. Brown, Phys. Rev. C 57, 3488 (1998).
- N. Frölich, H. Baier, and W. Bentz, Phys. Rev. C 57, 3447 (1998).
- F. de Jong and H. Lenske, Phys. Rev. C 57, 3099 (1998).
- I. Bombaci and U. Lombardo, Phys. Rev. C 44, 1892 (1991).
- W. Zuo, I. Bombaci, and U. Lombardo, Phys. Rev. C 60, 024605 (1999).
- F. Coestor, S. Cohen, B. D. Day, and C. M. Vincent, Phys. Rev. C 1, 769 (1970).
- B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997); Adv. Nucl. Phys. 16 1 (1986).
- R. Machleidt, Adv. Nucl. Phys. 16, 189 (1989); R. Brockmann and R. Machleidt, in *Nuclear Methods and Nuclear Equation of State*, Ed. M. Baldo (World Sci. Singapore (1999) p. 121.
- L. Sehn, C. Fuchs, and A. Faessler, Phys. Rev. C 56, 216 (1997).
- C. Fuchs, T. Waindzoch, A. Faessler, and D. S. Kosov, Phys. Rev. C 58, 2022 (1998).
- T. Gross-Boelting, C. Fuchs, and A. Faessler, Nucl. Phys. A 648, 105 (1999).
- G. E. Brown, W. Weise, G. Baym, and J. Speth, Comm. Nucl. Phys. 17, 39 (1987).
- M. Baldo, I. Bombaci, and G.F. Burgio, Astron. Astrophys. 328, 274 (1997).
- P. Gögelein, E. N. E. van Dalen, Kh. Gad, Kh. S. A. Hassaneen, and H. Müther, Phys. Rev. C 79, 024308 (2009).
- M. Baldo and Alaa Eldeen Shaban, Phys. Lett. B 661, 373 (2008).
- Z. H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, and H. R. Ma, Phys. Rev. C 74, 047304 (2006).
- V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C 49, 2950 (1994).
- M. Branco and J. Treiner, Nucl. Phys. A351,276 (1981).
- M. Baldo and L. S. Ferreira . Phys. Rev. C 59,694 (1999).
- M. Baldo and L. S. Ferreira . Phys. Rev. C 59,696 (1999).