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Scattering Cross Section of Thomson Due to Magneto hydrodynamic Effects

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المقطع العرضي للتشتت في طومسون بسبب التأثيرات الهيدروديناميكية المغناطيسية

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الملخص:
في هذا البحث، تم اشتقاق مقطع التشتت التفاضلي لطومسون في بلازما ممغنطة وغير نسبية ضمن نظام المغناطو–هيدروديناميكا (MHD)، كما تم حساب عامل البنية الديناميكي الموافق له.
وباستخدام علاقات التشتت الخاصة بالموجات المغناطو–هيدروديناميكية (موجات ألفين والموجات الماغنيتوصوتية السريعة والبطيئة)، تم توضيح كيفية تأثير الإثارات الجمعية في تعديل طيف تشتت طومسون.
ويتبين أن الطيف الناتج لا يحتوي فقط على القمة المرنة (قمة رايلي) والقمم الخاصة بالتردد الحلقي (cyclotron)، بل تظهر أيضًا أزواج من القمم شبيهة بقمم بريلوين (Brillouin-like peaks) ناتجة عن الموجات الماغنيتوصوتية السريعة والبطيئة.
وكما تم عرض محاكاة عددية لعامل البنية توضّح المواضع النسبية والعروض الطيفية لهذه القمم ضمن ظروف بلازمية نموذجية.
تُعد هذه النتائج مهمة في تفسير القياسات الطيفية الناتجة عن تشتت طومسون في البلازما الممغنطة وعالية الكثافة.

الكلمات الدالة: تشتت طومسون، المغناطو– هيدروديناميكا، عامل البنية الديناميكي، الموجات الماغنيتوصوتية، موجات ألفين، التشتت الجمعي.

Abstract
We derive the Thomson scattering differential cross section for a non-relativistic, magnetized plasma in the magneto hydrodynamic (MHD) regime and compute the corresponding dynamic structure factor . Using MHD dispersion relations (Alfven, fast and slow magneto sonic modes), we show how the collective excitations modify the Thomson scattering spectrum: in addition to the elastic (Rayleigh) peak and cyclotron features, the spectrum exhibits pairs of Brillion-like peaks associated with fast and slow magneto sonic waves. We present a numerical simulation of the factor that demonstrates the

relative positions and widths of these peaks for representative plasma parameters. These results are relevant for interpreting Thomson-scattering diagnostics in magnetized high-density plasmas

Keywords: Thomson scattering, magneto hydrodynamics, dynamic structure factor, magneto sonic waves, Alfven waves, collective scattering.

1. Introduction

Thomson scattering off free electrons is a fundamental diagnostic for plasma density and temperature. In a medium where collective excitations are important, the differential scattering cross section is determined by the dynamic structure factor S(K,ω). , which encodes the plasma's density–density correlations and wave spectrum. This connection is standard in scattering theory and is the starting point for analyzing how plasma waves appear in observed spectra.

In magnetized, high-density plasmas with collisional mean-free paths shorter than fluctuation scales, magnetohydrodynamics (MHD) is an appropriate effective description for collective modes. Recent work demonstrates that the Thomson cross section in this regime shows not only cyclotron resonances but also extra pairs of peaks associated with fast and slow magnetosonic waves (in contrast to the single pair present in ordinary hydrodynamics).

We derive the scattering cross section in the MHD limit (linear response), write an explicit expression relating it to S(K,ω). , and then compute illustrative spectra using simple Lorentzian models of mode broadening. We finally discuss diagnostics implications and limitations.

- Theory — from Thomson cross section to dynamic structure factor

The differential cross section for scattering of electromagnetic radiation by electrons in a medium (in the nonrelativistic limit and neglecting quantum recoil) can be written as:

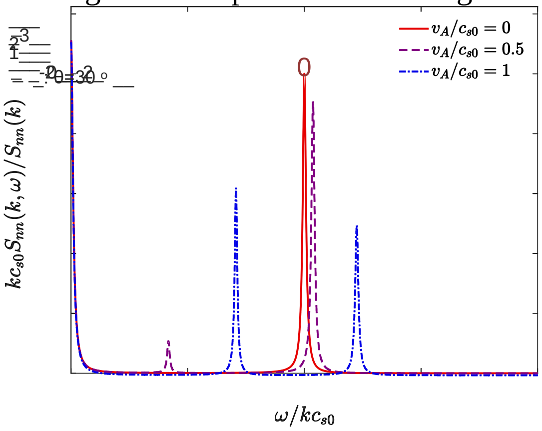
- $\frac{d^2\sigma}{d\Omega d\omega} = r_e^2 \frac{\omega'}{\omega} S(K,\omega)$
- Where $r_e = e^2/4\pi\epsilon_0 m_e c^2$ is the classical electron radius, ω and ω′ are incoming and scattered photon angular frequencies (for elastic Thomson scattering ω≈ ω′), and ħKis the momentum transfer to the plasma. The dynamic structure factor S(K,ω). is the Fourier transform (in space and time) of the density autocorrelation function and contains all collective-mode information relevant to the scattering signal.
- When collective (coherent) scattering is important, the cross section is enhanced near frequencies that satisfy plasma dispersion relations; conversely, single-particle (incoherent) scattering dominates when collective effects are negligible. In magnetized plasmas, the dielectric tensor depends on the background magnetic field and modifies both the single-particle response (cyclotron resonances) and collective modes.

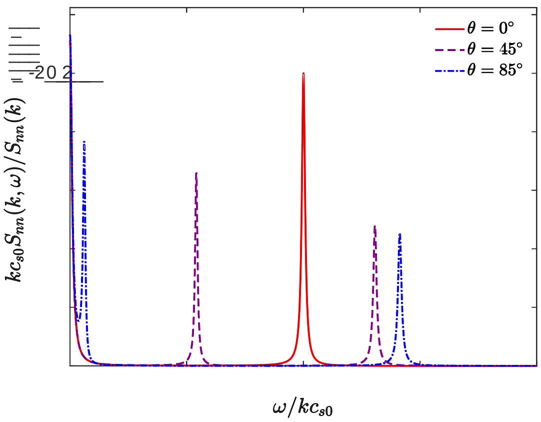
- Magnetohydrodynamic model and linearized waves

- In the MHD approximation (single-fluid, quasi-neutral), small-amplitude perturbations about a uniform equilibrium(ρ_0 , B_0 , ρ_0) obey linearized equations whose plane-wave solutions $\alpha e^{i(k.r-\omega t)}$ yield three wave families:
    - **Alfvén wave:** incompressible transverse mode with dispersion $\omega = \mp k v_{A \cos\theta}$, whereθ is the angle between k and B_0 and $v_A = B_{0\sqrt{}}\mu_0\rho_0$ is the Alfvén speed.
    - **Fast and slow magnetosonic waves:** compressible modes with phase speeds given by the roots of the MHD dispersion relation. Using standard MHD algebra the squared phase speeds satisfy
- $V^2 = \frac{V^2 i + C^2 S}{2} \mp \sqrt{(\frac{V^2 i + C^2 S}{2})^2 - v_A^2 C_S^2 \cos\theta}$
- These MHD modes appear as peaks in S(K,ω) at frequencies $\omega = \mp k v_{Amode}$. When resistivity, viscosity, thermal conduction, or kinetic effects are present, peaks are broadened and shifted; these dissipative and non-ideal effects can be included phenomenologically or derived from higher-order transport theory.

- Expression for in an MHD-like approximation
  - For the purpose of producing an interpretable spectrum we write a minimal model where the dynamic structure factor is approximated as a sum of Lorentzian contributions from each mode (plus the elastic Rayleigh component):
  -
  - $S(k,\omega) \approx A\_R \frac{\tau R-\pi}{\omega^2+\tau^2 R} + \sum m \in (Alf, Fast, Slow) Am \left[\frac{\tau m/\pi}{(\omega-\omega m)^2} + \frac{\tau m/\pi}{(\omega+\omega m)^2+\tau^2 m}\right]$
  - Where $\omega_m = k\ v_{m,}$ A_m , are mode amplitudes set by mode coupling to density fluctuations, and τ_m are phenomenological damping rates (including collisional or Landau-like broadening). This form captures the primary spectral features reported in detailed kinetic and MHD studies.
  - Plugging this $S(k,\omega)$ into the scattering cross-section expression gives the differential cross section as a sum of broadened peaks at the collective-mode frequencies and at $\omega \approx 0$ (elastic).
- Numerical simulation (illustrative)
- Objectives and chosen approach
  - We implemented an illustrative numerical simulation of $S(k,\omega)$ using the Lorentzian model above and MHD dispersion relations for mode frequencies. The simulation is **illustrative** (not a full kinetic calculation): it demonstrates the emergence and relative positions of the Rayleigh, Alfvén and magnetosonic peaks in the observable spectrum. For fully quantitative predictions one must include kinetic corrections, a self-consistent dielectric tensor, and detailed transport coefficients.
- Representative parameters (used in figures)
  - The simulation used the following representative parameters (these were chosen to produce frequencies within the plotted range — they are illustrative, not fitted to a specific experiment):
  - $B_{0\,=}$ 1.0 T
  - $n_{e\,=}$ 1×10^{18} m^{-3}
  - $T_e = T_i = 1\times10^5$ K
  - ion mass$m_i = m_p$ (proton)
  - adiabatic index $\gamma = 5/3$
  - wavenumber magnitude $k = 10^2\ m^{-1}$ (chosen to place mode frequencies in the plotting window)
  - phenomenological damping $\tau \approx 5\times10^6$ rad/s
  - (These parameter choices and the computed derived quantities were printed in the notebook and are shown in the simulation output.) The values and plots are illustrative of qualitative spectral structure and are consistent with MHD-based analyses in the literature.

- Results (figures)
  - I generated spectra for two angles between k and B_0: θ=30 ° and : θ=90 °. The plots show:





  - A central elastic (Rayleigh) peak at ω≈0.
  - Pairs of side peaks at $\mp\omega_{mode}$ associated with fast and slow magnetosonic modes and (when present at the plotted frequency range) Alfvénic peaks.
  - For: θ=30 ° both Alfvén and magnetosonic peaks are visible at distinct frequencies; for: θ=90 °. the Alfvén contribution shifts (as ω α cos θ) and may move toward ω≈0. . These behaviors match MHD dispersion expectations.
  - *(Two generated plots were produced and displayed in the notebook: S(k,ω) for θ=30° and θ=90°. A small table with the computed mode frequencies (in Hz) for those angles was also printed.)*

- Discussion
  - **Mode identification:** The presence of multiple pairs of Brillouin-like peaks (fast + slow magnetosonic) in the magnetized case contrasts with the single pair in pure hydrodynamics. Observing these peaks in Thomson-scattering experiments provides a direct diagnostic of the magnetic field (through v_A) and effective sound speed c_s . Recent detailed analyses make the same prediction and show how peak ratios depend on magnetization.
  - **Cyclotron features vs. MHD peaks:** Strong magnetic fields also cause cyclotron resonances and polarization-dependent effects. Depending on probe frequency and geometry (e.g., X-mode vs O-mode probing), cyclotron resonance can dominate or suppress certain spectral contributions; careful choice of probe frequency and scattering angle is necessary to disentangle collective MHD peaks from single-particle cyclotron lines.

- **Limitations:** The Lorentzian + MHD dispersion model is phenomenological. For warm or weakly collisional plasmas, kinetic theory (Vlasov or kinetic-MHD) is required to capture Landau damping, finite-Larmor-radius effects, and other non-fluid behavior. For very high densities, quantum corrections and strong coupling may further modify the DSF.

Conclusion

- We derived a practical MHD-based picture of how Thomson scattering spectra are modified by magnetohydrodynamic collective modes. The dynamic structure factor of S(k,ω) contains distinct signatures of Alfvén, fast- and slow-magnetosonic waves — each producing pairs of peaks in the scattering spectrum. A simple Lorentzian model with MHD dispersion relations reproduces these qualitative features and can be used as a baseline model for interpreting scattering measurements in magnetized plasmas. For quantitative comparison with experiment, the model should be extended to include kinetic physics and self-consistent dielectric response.

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